Geophysical Research Abstracts
Vol. 15, EGU2013-4620, 2013
EGU General Assembly 2013

To the Theory of a Hydraulic Jump, Rolling Waves and Transformation of Long Waves on Flows of Variable Depth

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Possible approaches to the linearization of a system of basic equations of long waves on stationary flows of arbitrary depth are discussed. It is shown that the accurately fulfilled linearization of this system makes it possible to obtain the following one-dimensional differential equation for long wave disturbances:

$$\varphi_{tt} + 2U_0\varphi_{xt} + \left(\frac{3}{2}U_0^2 - gh\right)\varphi_{xx} + U_{0x}\varphi_t + (3U_0U_{0x} - gh_x)\varphi_x = g\,(U_0h)_x - \frac{3}{2}U_0^2U_{0x} \quad , \tag{1}$$

where $U_0\,(x)$ is a given stationary component of the stream velocity; $\varphi(x,t)$ is the sought function which is an analogue of the velocity potential of wave disturbances ($u(x,t) = \varphi_x$); $h(x)$ is the variable depth of water in a statically undisturbed state; g is the gravity acceleration; t is the time; the $x-$axis is superposed on the undisturbed free surface.

For a classical problem of motion of long waves on flows of constant depth ($h = const$) and velocity ($U_0 = const$) this equation leads to the following formula for the phase velocity of a long wave:

$$c = U_0 \pm \sqrt{gh - \frac{U_0^2}{2}} \quad , \tag{2}$$

where the sign „+“ before the square root corresponds to waves directed along the flow and the sign „-“ to waves directed counter the flow. Relation (**??**) essentially differs from the classical Lagrange formula

$$c = U_0 \pm \sqrt{gh} \tag{3}$$

and allows us to formulate the criteria of formation of a hydraulic jump, which is a blocked wave, as well as of rolling long discontinuous waves on rapid streams without using the momentum change equation. In particular, (**??**) implies that waves cannot move against the flow, i.e. $c = 0$ when $U_0^2 = gh$ or when the Froude number is $Fr_0 = U_0^2/gh \geq 1$, whereas (**??**) implies that $c = 0$ when $Fr_0 \geq 2/3$. Along with this, according to (**??**), the wave blocking condition precedes the wave destruction condition $Fr_0 \geq 2$ for which the subroot expression in (**??**) becomes negative. In that case, the oscillation frequency takes a complex value and the solution brings to Helmholtz instability.

Thus, in the framework of (**??**), we can give a good explanation of the fact confirmed by experiments (Chertousov 1962) that near the unit value of the Froude number or, speaking more exactly, when $\frac{2}{3} \leq Fr_0 \leq 2$, the hydraulic jump, which is actually a blocked long wave, is the so-called undular jump.

A perfect hydraulic jump occurs only for $Fr_0 > 2$, while for $Fr_0 < 2/3$ a long wave freely moves against the flow, i.e., a flow of large depth (a long wave) overlaps a calm flow of small depth without wave destruction.

Considering a hydraulic jump as a blocked long wave, we derive from formula (**??**), without using the momentum equation, a simple formula for calculating conjugate depths of a hydraulic jump:

$$\frac{h_2}{h_1} = \sqrt{\frac{3}{2}\frac{U_1^2}{gh_1}} \approx 1,225\sqrt{Fr_1} \tag{4}$$

which, as different from Belanger's formula commonly used in the hydraulics courses, is applicable both to „an undular jump“ and to „a perfect jump“.

For waves on smoothly changing flows, asymptotic solutions are obtained by the WKB method.

References

1. Chertousov M.D. Hydraulics. A Special Course. Gosenergoizdat, Moscow-Leningrad, 1962, 630 p. (in Russian)
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