A development of projecting operators technique in AGW theory

Sergey Leble and Anna Perelomova

Gdansk University of Technology, Technical physics and applied mathematics, Differential equations and applied mathematics, Gdansk, Poland (leble@mif.pg.gda.pl)

This paper develops the ideas of [1] in a spirit of recent realization [2] in which main attention is paid to acoustics and entropy mode co-existence and energetics. Now we include internal waves, having in mind general problem of geophysical hydrodynamics, neglecting Earth rotation terms (for the terms account possibility see [1]).

The relations connecting perturbations specific for gravity, acoustic and entropy modes in a fluid affected by constant mass force, are derived. They allow to decompose the total vector of perturbations "a" and the overall energy into acoustic, gravity and non-wave (zero frequency) parts uniquely at any instant. In order to do this, five quantities are required, for example total perturbations in entropy, pressure and velocity vector. The corresponding projecting operators $P(i)$, i=1,2,3,4,5 allow to specify the contributions (gravity, acoustic and entropy modes) in local measurements of the total perturbations, providing a geophysical diagnostics. Measurements of the basic parameters perturbations field may be also decomposed into the modes and, therefore, the problem of initialization is solved. Such operators are built as integral ones by aid of Fourier transformation either in space for initial problem or in space-time for a boundary problem.

The method allows direct applications to problems with external sources vector "f" by the following scheme. The linearized basic equation of geophysical hydrodynamics of the form $da/dt - La = f$ is decomposed by application of the projecting operators $P(i)$ that commute with the evolution operator "L" by definition, hence $(d/dt - L)P(i)a = P(i)f$. Such form is equivalent to a variables change (substitution $a = (P(1) + ... + P(5))a$) which is one-to-one map because the set of projecting operators is complete $P(1) + ... + P(5) = I$. The technique is applied to nonlinear models in similar way: $(d/dt - L)P(i) a = P(i)N(a)$, where $N(a)$ are nonlinear terms of the basic system [1].

An important problem of energy balance in the atmosphere needs the appropriate definitions and decomposition. The evaluations are made in regard to the content of gravity, acoustic and non-wave parts of the total energy functional. The functional division into modes additives allows a constitutive definitions and follow the energy exchange between waves and its cumulation in entropy mode.

1. S.B. Leble, Nonlinear Waves in Waveguides with Stratification, Springer-Verlag, Berlin, 1990.
2. S.Leble, A. Perelomova. Problem of proper decomposition and initialization of acoustic and entropy modes in a gas affected by the mass force. AMM, 37(2013) 629-635.