



Estimating extreme environmental variables by POT methods: behavior of Maximum Likelihood Estimators and distinction between statistical threshold and location parameter

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For a decade, the methodology for determining extreme values of environmental variables has converged towards the so-called GPD-Poisson model. The Peaks-Over-Threshold (POT) method is used for extracting extreme i.i.d. data from a time series. Excesses above a statistical meaningful threshold are fit to a 2-parameter Generalized Pareto Distribution (GPD), often with the Maximum Likelihood Estimator (MLE). Last, extreme values for desired return periods (quantiles) and confidence intervals are derived. In this approach, the GPD plays the role of an asymptotic approximation of the true law of excesses; hence it is valid when the threshold is high enough: a correct determination of the threshold value is thus crucial. Besides, a close examination of the sensitivity of the estimations with respect to the threshold shows that the use of 2-parameters (scale and shape) distributions yields unstable results. Considering the GPD (results are similar with other distributions), we explain in this presentation the need for a location parameter, distinct from the threshold, and the non-optimality of ML estimators.

When the threshold is allowed to vary between two consecutive data values of the extreme sample, the ML estimates of the scale and shape parameters of the distribution do not remain constant. In other words, a slight translation of the sample may lead to a significant change in the estimated quantiles, although the same physical events are extrapolated. The likelihood of the 2-parameter GPD model is continuously increasing when the threshold tends to the first value of the sample of the excesses, with non-null derivatives. When extending the model to a 3-parameter GPD (adding a location parameter), it can be shown that the maximum likelihood is reached at the open upper bound of the interval of validity of this parameter, with non-null derivatives. Still, the asymptotic properties of the MLE require that the maximum be reached on an interior point of an open set. Thus the widely spread 2-parameter GPD-Poisson model makes a confusion between the threshold, whose role should be limited to data selection/censorship, and the location parameter, whose role is to accurately set the origin of the distribution. Hence it is necessary to fit 3-parameter distributions and MLE shouldn't be used since its properties are not proven. The L-moments estimator, used for comparison purposes, allows a proper estimation of the 3 parameters and yields estimated quantiles remaining constant when the threshold varies between two data values.