



## Simulation of wind wave growth with reference source functions

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We present results of extensive simulations of wind wave growth with the so-called reference source function in the right-hand side of the Hasselmann equation written as follows

$$\frac{\partial N_{\mathbf{k}}}{\partial t} + \nabla_{\mathbf{k}} \omega \cdot \nabla_{\mathbf{x}} N_{\mathbf{k}} = S_{nl} + S_{in} + S_{diss} \quad (1)$$

First, we use Webb's algorithm [8] for calculating the exact nonlinear transfer function  $S_{nl}$ . Second, we consider a family of wind input functions in accordance with recent consideration [9]

$$S_{in} = \gamma(\mathbf{k}) N_{\mathbf{k}}, \quad \gamma(\mathbf{k}) = \gamma_0 \omega \left( \frac{\omega}{\omega_0} \right)^s f_{in}(\theta). \quad (2)$$

Function  $f_{in}(\theta)$  describes dependence on angle  $\theta$ . Parameters in (2) are tunable and determine magnitude (parameters  $\gamma_0, \omega_0$ ) and wave growth rate  $s$  [9]. Exponent  $s$  plays a key role in this study being responsible for reference scenarios of wave growth:  $s = 4/3$  gives linear growth of wave momentum,  $s = 2$  – linear growth of wave energy and  $s = 8/3$  – constant rate of wave action growth. Note, the values are close to ones of conventional parameterizations of wave growth rates (e.g.  $s = 1$  for [7] and  $s = 2$  for [5]).

Dissipation function  $S_{diss}$  is chosen as one providing the Phillips spectrum  $E(\omega) \sim \omega^5$  at high frequency range [3] (parameter  $\omega_{diss}$  fixes a dissipation scale of wind waves)

$$S_{diss} = C_{diss} \mu_w^4 \omega N(\mathbf{k}) \Theta(\omega - \omega_{diss}) \quad (3)$$

Here frequency-dependent wave steepness

$$\mu_w^2 = E(\omega, \theta) \omega^5 / g^2$$

makes this function to be heavily nonlinear and provides a remarkable property of stationary solutions at high frequencies: the dissipation coefficient  $C_{diss}$  should keep certain value to provide the observed power-law tails close to the Phillips spectrum  $E(\omega) \sim \omega^{-5}$ . Our recent estimates [3] give  $C_{diss} \approx 2.0$ .

The Hasselmann equation (1) with the new functions  $S_{in}, S_{diss}$  (2,3) has a family of self-similar solutions of the same form as previously studied models [1,3,9] and proposes a solid basis for further theoretical and numerical study of wave evolution under action of all the physical mechanisms: wind input, wave dissipation and nonlinear transfer.

Simulations of duration- and fetch-limited wind wave growth have been carried out within the above model setup to check its conformity with theoretical predictions, previous simulations [2,6,9], experimental parameterizations of wave spectra [1,4] and to specify tunable parameters of terms (2,3). These simulations showed realistic spatio-temporal scales of wave evolution and spectral shaping close to conventional parameterizations [e.g. 4]. An additional important feature of the numerical solutions is a saturation of frequency-dependent wave steepness  $\mu_w$  in short-frequency range.

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