



Simulation of wind wave growth with reference source functions

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We present results of extensive simulations of wind wave growth with the so-called reference source function in the right-hand side of the Hasselmann equation written as follows

$$\frac{\partial N_{\mathbf{k}}}{\partial t} + \nabla_{\mathbf{k}} \omega \cdot \nabla_{\mathbf{x}} N_{\mathbf{k}} = S_{nl} + S_{in} + S_{diss} \quad (1)$$

First, we use Webb's algorithm [8] for calculating the exact nonlinear transfer function S_{nl} . Second, we consider a family of wind input functions in accordance with recent consideration [9]

$$S_{in} = \gamma(\mathbf{k}) N_{\mathbf{k}}, \quad \gamma(\mathbf{k}) = \gamma_0 \omega \left(\frac{\omega}{\omega_0} \right)^s f_{in}(\theta). \quad (2)$$

Function $f_{in}(\theta)$ describes dependence on angle θ . Parameters in (2) are tunable and determine magnitude (parameters γ_0, ω_0) and wave growth rate s [9]. Exponent s plays a key role in this study being responsible for reference scenarios of wave growth: $s = 4/3$ gives linear growth of wave momentum, $s = 2$ – linear growth of wave energy and $s = 8/3$ – constant rate of wave action growth. Note, the values are close to ones of conventional parameterizations of wave growth rates (e.g. $s = 1$ for [7] and $s = 2$ for [5]).

Dissipation function S_{diss} is chosen as one providing the Phillips spectrum $E(\omega) \sim \omega^5$ at high frequency range [3] (parameter ω_{diss} fixes a dissipation scale of wind waves)

$$S_{diss} = C_{diss} \mu_w^4 \omega N(\mathbf{k}) \Theta(\omega - \omega_{diss}) \quad (3)$$

Here frequency-dependent wave steepness

$$\mu_w^2 = E(\omega, \theta) \omega^5 / g^2$$

makes this function to be heavily nonlinear and provides a remarkable property of stationary solutions at high frequencies: the dissipation coefficient C_{diss} should keep certain value to provide the observed power-law tails close to the Phillips spectrum $E(\omega) \sim \omega^{-5}$. Our recent estimates [3] give $C_{diss} \approx 2.0$.

The Hasselmann equation (1) with the new functions S_{in}, S_{diss} (2,3) has a family of self-similar solutions of the same form as previously studied models [1,3,9] and proposes a solid basis for further theoretical and numerical study of wave evolution under action of all the physical mechanisms: wind input, wave dissipation and nonlinear transfer.

Simulations of duration- and fetch-limited wind wave growth have been carried out within the above model setup to check its conformity with theoretical predictions, previous simulations [2,6,9], experimental parameterizations of wave spectra [1,4] and to specify tunable parameters of terms (2,3). These simulations showed realistic spatio-temporal scales of wave evolution and spectral shaping close to conventional parameterizations [e.g. 4]. An additional important feature of the numerical solutions is a saturation of frequency-dependent wave steepness μ_w in short-frequency range.

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