



A non-linear characterisation of state dependency in the gradient of the output for forecasting

Paul Smith, Keith Beven, and Peter Young

Lancaster University, Lancaster Environment Centre, Lancaster, United Kingdom (p.j.smith@lancs.ac.uk)

In hydrology a discrete time series of observations (e.g. water level or discharge) can often be effectively modeled from an input series (e.g. rainfall or an upstream water level) which may be lagged to generate forecast lead time. Much success has been achieved using Hammerstein models consisting of a non-linear transform of the input data followed by filtering using a linear transfer function. One advantage that arises in the implementation of such models is that when the non-linear transform is computed from observed data linear filtering can be applied to assimilate data and improve forecasts. In many the Hammerstein model can be interpreted in a physically meaningful fashion thus shown to be Data Based Mechanistic (Young, *Environmetrics* 5, 1994). This can be a useful step in both gaining acceptance of the techniques used but also in relating data analysis and physical theory.

There is increasing evidence that in some systems the Hammerstein structure outlined cannot adequately capture the dynamics witnessed. This has lead to a number of alternative propositions such as transfer functions where all the parameters are dependent upon some external series of data (Laurain et al., *Automatica* 46(6), 2010). Such models are though difficult to interpret from a physical perspective and require the selection of an appropriate external series. An alternative modelling strategy is outlined and contrasted with the Hammerstein form. This is based on the characterisation of the gradient of the response variable dependent upon its state and the input at a given time. The methodology is applied to model water levels on a tidal river. The application of filtering to improve the forecasts by assimilating recent observations is illustrated. It is shown that not only does the method produce good forecasts but also offers a meaningful physical interpretation.