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Statistics of extreme waves in the framework of one-dimensional Nonlinear Schrodinger Equation

Dmitry Agafontsev (1) and Vladimir Zakharov (2)

(1) P.P. Shirshov Institute of Oceanology, Moscow, Russian Federation (Dmitry.Agafontsev@gmail.com), (2) Department of Mathematics, University of Arizona, Tucson, USA (zakharov@math.arizona.edu)

We examine the statistics of extreme waves for one-dimensional classical focusing Nonlinear Schrodinger (NLS) equation,

$$i\Psi_t + \Psi_{xx} + |\Psi|^2\Psi = 0, \qquad (1)$$

as well as the influence of the first nonlinear term beyond Eq. (1) - the six-wave interactions - on the statistics of waves in the framework of generalized NLS equation accounting for six-wave interactions, dumping (linear dissipation, two- and three-photon absorption) and pumping terms,

$$i\Psi_t + (1-id_l)\Psi_{xx} + (1+id_{2p})|\Psi|^2\Psi + (\alpha+id_{3p})|\Psi|^4\Psi = ip\Psi, \qquad (2)$$
$$\alpha, d_l, d_{2p}, d_{3p}, p \ll 1, \quad d_{3p} \ll \alpha.$$

We solve these equations numerically in the box with periodically boundary conditions starting from the initial data $\Psi_{t=0} = F(x) + \epsilon(x)$, where $F(x)$ is an exact modulationally unstable solution of Eq. (1) seeded by stochastic noise $\epsilon(x)$ with fixed statistical properties. We examine two types of initial conditions $F(x)$: (a) condensate state $F(x) = 1$ for Eq. (1)-(2) and (b) cnoidal wave for Eq. (1). The development of modulation instability in Eq. (1)-(2) leads to formation of one-dimensional wave turbulence. In the integrable case the turbulence is called integrable and relaxes to one of infinite possible stationary states. Addition of six-wave interactions term leads to appearance of collapses that eventually are regularized by the dumping terms. The energy lost during regularization of collapses in (2) is restored by the pumping term. In the latter case the system does not demonstrate relaxation-like behavior.

We measure evolution of spectra $I_k =< |\Psi_k|^2 >$, spatial correlation functions and the PDFs for waves amplitudes $|\Psi|$, concentrating special attention on formation of "fat tails" on the PDFs. For the classical integrable NLS equation (1) with condensate initial condition we observe Rayleigh tails for extremely large waves and a "breathing region" for middle waves with oscillations of the frequency of waves appearance with time, while nonintegrable NLS equation with dumping and pumping terms (2) with the absence of six-wave interactions $\alpha = 0$ demonstrates perfectly Rayleigh PDFs without any oscillations with time. In case of the cnoidal wave initial condition we observe severely non-Rayleigh PDFs for the classical NLS equation (1) with the regions corresponding to 2-, 3- and so on soliton collisions clearly seen of the PDFs. Addition of six-wave interactions in Eq. (2) for condensate initial condition results in appearance of non-Rayleigh addition to the PDFs that increase with six-wave interaction constant α and disappears with the absence of six-wave interactions $\alpha = 0$.

References:

[1] D.S. Agafontsev, V.E. Zakharov, *Rogue waves statistics in the framework of one-dimensional Generalized Nonlinear Schrodinger Equation*, arXiv:1202.5763v3.