Geophysical Research Abstracts
Vol. 15, EGU2013-6520, 2013
EGU General Assembly 2013
© Author(s) 2013. CC Attribution 3.0 License.

A research of wave equation of shallow water with sediment on inclined channel

Muneyuki Arai
Meijo University, Civil Engineering, Nagoya, Japan (arai@meijo-u.ac.jp, +81 52-832 1178)

Viscous debris flow in China is typical intermittent flow with high concentrated sediment. These intermittent surges flows are observed not only in China of viscous flow but also in the European Alps and other mountains region. It is important to obtain wave equation for wave motion of intermittent surges. This research shows wave equation of shallow water with sediment on inclined channel which include intermittent debris flow. Using non-dimensional basic equation as follows, Laplace equation,

$$\frac{\partial^2\phi'}{\partial x'^2}+\frac{\partial^2\phi'}{\partial y'^2}=0 \tag{1}$$

bottom boundary condition,

$$\frac{\partial\phi}{\partial y'}=0,\quad (y'=-1;\text{bottom of mean depth } h_0) \tag{2}$$

surface condition (conservation condition of flow surface),

$$-\frac{\partial\phi'}{\partial y'}+\frac{\partial\eta'}{\partial t'}+\frac{\partial\phi'}{\partial t'}\frac{\partial\eta'}{\partial x'}=0,\quad (y'=0;\text{surface of mean depth } h_0) \tag{3}$$

and equation of momentum,

$$\frac{\partial\phi'}{\partial t'}+\frac{1}{2}\left(\frac{\partial\eta'}{\partial x'}\right)^2-{c_0}'\tan\theta\cdot x'+{c_0}'^2\left(1+\eta'\right)+\tan\theta\frac{{c_0}'}{{u_0}'}\phi'=0 \tag{4}$$

where, $\phi=\phi(x,y,t)$: potential function, $\phi'=\frac{\phi}{h_0\,v_{p0}}$, x : coordinate axis of flow direction, $x'=\frac{x}{h_0}$, y : coordinate axis of depth direction, $y'=\frac{y}{h_0}$, h_0 : mean depth, t : time, $t'=t\frac{v_{p0}}{h_0}$, v_{p0} : velocity parameter in G-M transfer, u_0 : mean velocity, ${u_0}'=\frac{u_0}{v_{p0}}$, $h=h_0+\eta$: depth of flow, η : deflection from h_0, $\eta'=\frac{\eta}{h_0}$, g : acceleration due to gravity, θ : slope angle of the channel, $c_0=\sqrt{gh_0\cos\theta}$, ${c_0}'=\frac{c_0}{v_{p0}}$.

Using a method of perturbation, Gardner-Morikawa(G-M) transfer, $\xi'=\epsilon^{\frac{1}{2}}\left(x'-t'\right)$, $\tau'=\epsilon^{\frac{3}{2}}$ and ϵ =perturbation parameter, the wave equation is obtained as follows,

$$\frac{\partial\eta'}{\partial\tau'}+\frac{1}{2}\left(1+2{c_0}'^2\right)\eta'\frac{\partial\eta'}{\partial\xi'}-\frac{1}{2}\tan\theta\frac{{c_0}'^2}{{u_0}'}\frac{\partial^2\eta'}{\partial\xi'^2}+\frac{1}{2}\left(\frac{1}{{c_0}'^2}-1\right)\frac{\partial^3\eta'}{\partial\eta'^3}=0 \tag{5}$$

and in case of $v_{p0}=\sqrt{gh_0\cos\theta}=c_0$, the wave equation is as follow,

$$\frac{\partial\eta'}{\partial\tau'}+\frac{3}{2}\eta'\frac{\partial\eta'}{\partial\xi'}-\frac{1}{2}\tan\theta\frac{\partial^2\eta'}{\partial\xi'^2}=0 \tag{6}$$

This is a Burgars equation. These mathematical considerations indicate the monitoring system of intermittent debris flow which should observe the mean velocity and wave velocity individually.