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Regularized Solutions for Very Large Linear Systems from Geotomography

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We propose a mathematical approach to approximately obtain regularized solutions to very large linear systems.

In inverse problems from Geotomography one often encounters linear systems of very large size. Thus, we are faced with the task of solving a linear system $Ax = b$ with $A \in \mathbb{R}^{m \times N}$ where both m and N may be in the millions. Since A is typically not well conditioned, the right hand side vector b contains noise and there is typically less data than unknowns ($m < N$), regularization is required in terms of constraints put on the solution x. In the most commonly used case, one puts a restriction on the two norm of x resulting in a least squares problem with constraints such as:

$\min ||Ax - b||_2$ s.t. $||x||_2 < \alpha$ or $\min ||x||_2$ s.t. $||Ax - b||_2 < \beta$

Both of these can be reformulated into the unconstrained minimization problem:

$\bar{x} = \arg\min_x \left[||Ax - b||_2^2 + \lambda ||x||_2^2 \right]$ for some $\lambda \in \mathbb{R}$ with $\lambda > 0$. The solution to this quadratic problem is obtained via the linear system $(A^T A + \lambda I)\bar{x} = A^T b$ whose solution can be approximated by means of a number of iterative algorithms such as conjugate gradients. However, problems arise when the matrix A is very large and cannot be easily stored in memory.

In our approach, we begin by approximating matrix vector operations with the matrix A using a substantially smaller compressed matrix $M = \text{Thr}(AW_c^T)$ obtained via a wavelet transform and thresholding of the rows of A. This allows us to approximate matrix-vector operations with the big matrix using the smaller matrix M:

Ax $\approx MW_c^{-T} x$ and $A^T y \approx W_c^{-1} M^T y$

Using these relations and a fast randomized algorithm we proceed to construct the low rank SVD approximation of A: A $\approx U_k \Sigma_k V_k^T$ with U_k being $m \times k$, Σ_k being $k \times k$, and V_k being $N \times k$ with $k << min(m, N)$. We then apply this approximation to efficiently construct regularized solutions, without iteration. Finally, we discuss the process for very large matrices where we first break up the matrix A into several sub matrices $A_1, A_2, \ldots, A_P$, compute the low rank SVD of the individual submatrices and use this information to approximate the solution with the full matrix.