



Variance of Dispersion Coefficients in Heterogeneous Porous Media

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We study the dispersion of a passive solute in heterogeneous porous media using a stochastic modeling approach. Heterogeneity on one hand leads to an increase of solute spreading, which is described by the well-known macrodispersion phenomenon. On the other hand, it induces uncertainty about the dispersion behavior, which is quantified by ensemble averages over suitably defined dispersion coefficients in single medium realizations. We focus here on the sample to sample fluctuations of dispersion coefficients about their ensemble mean values for solutes evolving from point-like and extended source distributions in $d = 2$ and $d = 3$ spatial dimensions. The definition of dispersion coefficients in single medium realizations for finite source sizes is not unique, unlike for point-like sources. Thus, we first discuss a series of dispersion measures, which describe the extension of the solute plume, as well as dispersion measures that quantify the solute dispersion relative to the injection point. The sample to sample fluctuations of these observables are quantified in terms of the variance with respect to their ensemble averages. We find that the ensemble averages of these dispersion measures may be identical, their fluctuation behavior, however, may be very different. This is quantified using perturbation expansions in the fluctuations of the random flow field. We derive explicit expressions for the time evolution of the variance of the dispersion coefficients. The characteristic time scale for the variance evolution is given by the typical dispersion time over the characteristic heterogeneity scale and the dimensions of the source. We find that the dispersion variances asymptotically decrease to zero in $d = 3$ dimensions, which means, the dispersion coefficients are self-averaging observables, at least for moderate heterogeneity. In $d = 2$ dimensions, the variance converges towards a finite asymptotic value that is independent of the source distribution. Dispersion is not self-averaging, which may be traced back to the lesser sampling efficiency in $d = 2$. This work sheds some new light on the concepts of dispersion in single medium realizations and their quantification in terms of ensemble averages in a stochastic modeling framework. These findings may be relevant for the interpretation of dispersion data from field and laboratory experiments.