



Evaluation of kurtosis of JONSWAP spectra

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Typical random wind waves in the sea are characterized by broad-band spectra and quasi-Gaussian statistics. While the evolution of wave field spectra in the ocean is well studied, very little is known about how departure of wave statistics from Gaussian depends on characteristics of wave spectra. This information is needed for many applications but is very difficult to extract from observations outside laboratory. It is common to characterize the departure of wave statistics from Gaussianity by value of kurtosis, a fourth-order statistical moment. Non-zero values of kurtosis mean an increased or decreased probability of extreme waves (compared to that in a Gaussian sea), which is important for assessing the risk of freak waves and other applications. For quasi-Gaussian waves there are two contributions to kurtosis. The first one, $C_4^{(b)}$, is due to bound harmonics, while the second one, “dynamic kurtosis” $C_4^{(d)}$, is linked to nonlinear wave-wave interactions. Under standard weak turbulence assumptions Janssen (2003) derived expressions for both components of kurtosis in terms of energy spectra. However, since the evaluation of the resulting 6-dimensional integrals is technically challenging, it has not been implemented for any experimental wave spectra.

Here we evaluate $C_4^{(d)}$ and $C_4^{(b)}$ for the JONSWAP spectra, a widely used family of parametrisations of the observed spectra. We choose the k -form of the JONSWAP spectrum with the peak at $k = 1$. The frequency spectra are considered in the range $0.5 < \omega < 3$. The magnitude of the spectra is specified by parameter α , where α is proportional to the square of the steepness. The range of α corresponds to the range of steepness from 0.04 to 0.3. The JONSWAP parameter γ characterizing “peakedness” of the spectra. γ is taken from 1 to 10. Angular distributions of the $(\cos \theta)^N$ type are considered for several values of N .

Thus we find behaviour of both components of kurtosis in the three-dimensional parameter space (α, γ, N) and their sensitivity to approximations of the spectral shape. This provides a good idea on the degree of departure of wave statistics from Gaussian for realistic wave fields.