



Resilience and vulnerability to a natural hazard: A mathematical framework based on viability theory

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This deals with the response of a coupled human and natural system (CHANS) to a natural hazard by using the concepts of resilience and vulnerability within the mathematical framework of viability theory. This theory applies to time-evolving systems such as CHANS and assumes that their desirable properties can be defined as a subset of their state space. Policies can also apply to influence the dynamics of such systems: viability theory aims at finding the policies which keep the properties of a controlled dynamical system for so long as no disturbance hits it. The states of the system such that the properties are guaranteed constitute what is called the viability kernel. This viability framework has been extended to describe the response to a perturbation such as a natural hazard. Resilience describes the capacity of the CHANS to recover by getting back in the viability kernel, where its properties are guaranteed until the onset of the next major event. Defined for a given controlled trajectory that the system may take after the event ends, resilience is (a) whether the system comes back to the viability kernel within a given budget such as a time constraint, but also (b) a decreasing function of vulnerability. Computed for a given trajectory as well, vulnerability is a measure of the consequence of violating a property. We propose a family of functions from which cost functions and other vulnerability indicators can be derived for a certain trajectory. There can be several vulnerability functions, representing for instance social, economic or ecological vulnerability, and each representing the violation of an associated property, but these functions need to be ultimately aggregated as a single indicator. Computing the resilience and vulnerability of a trajectory enables the viability framework to describe the response of both deterministic and stochastic systems to hazards. In the deterministic case, there is only one response trajectory for a given action policy, and methods exist to find the actions which yield the most resilient trajectory, namely the least vulnerable trajectory for which recovery is complete. In the stochastic case however, there is a range of possible trajectories. Statistics can be derived from the probability distribution of the resilience and vulnerability of the trajectories. Dynamic programming methods can then yield either the policies that maximize the probability of being resilient by achieving recovery within a given time horizon, or those which minimize a given vulnerability statistic. These objectives are different and can be in contradiction, so that trade-offs may have to be considered between them. The approach is illustrated in both the deterministic and stochastic cases through a simple model of lake eutrophication, for which the desirable ecological properties of the lake conflict with the economic interest of neighboring farmers.