



## **Nonlinear storage models of unconfined flow through a shallow aquifer on an inclined base and their quasi-steady flow application**

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Hillslope processes involving flow through an inclined shallow aquifer range from subsurface stormflow to stream base flow (drought flow, or groundwater recession flow). In the case of recharge, the infiltrating water moves vertically as unsaturated flow until it reaches the saturated groundwater, where the flow is approximately parallel to the base of the aquifer. Boussinesq used the Dupuit-Forchheimer (D-F) hydraulic theory to formulate unconfined groundwater flow through a soil layer resting on an impervious inclined bed, deriving a nonlinear equation for the flow rate that consists of a linear gravity-driven component and a quadratic pressure-gradient component. Inserting that flow rate equation into the differential storage balance equation (volume conservation) Boussinesq obtained a nonlinear second-order partial differential equation for the depth. So far however, only few special solutions have been advanced for that governing equation. The nonlinearity of the equation of Boussinesq is the major obstacle to deriving a general analytical solution for the depth profile of unconfined flow on a sloping base with recharge (from which the discharges could be then determined). Henderson and Wooding (1964) were able to obtain an exact analytical solution for steady unconfined flow on a sloping base, with recharge, and their work deserves special note in the realm of solutions of the nonlinear equation of Boussinesq. However, the absence of a general solution for the transient case, which is of practical interest to hydrologists, has been the motivation for developing approximate solutions of the non-linear equation of Boussinesq.

In this work, we derive the aquifer storage function by integrating analytically over the aquifer base the depth profiles resulting from the complete nonlinear Boussinesq equation for steady flow. This storage function consists of a linear and a nonlinear outflow-dependent term. Then, we use this physics-based storage function in the transient storage balance over the hillslope, obtaining analytical solutions of the outflow and the storage, for recharge and drainage, via a quasi-steady flow calculation. The hydraulically derived storage model is thus embedded in a quasi-steady approximation of transient unconfined flow in sloping aquifers. We generalise this hydrologic model of groundwater flow by modifying the storage function to be the weighted sum of the linear and the nonlinear storage terms, determining the weighting factor objectively from a known integral quantity of the flow (either an initial volume of water stored in the aquifer or a drained water volume). We demonstrate the validity of this model through comparisons with experimental data and simulation results.