



## **Lake eutrophication and environmental change: A viability framework for resilience, vulnerability and adaptive capacity**

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We present a simple stochastic model of lake eutrophication to demonstrate how the mathematical framework of viability theory fosters operational definitions of resilience, vulnerability and adaptive capacity, and then helps understand which response one should bring to environmental changes. The model represents the phosphorus dynamics, given that high concentrations trigger a regime change from oligotrophic to eutrophic, and causes ecological but also economic losses, for instance from tourism. Phosphorus comes from agricultural inputs upstream of the lake, and we will consider a stochastic input. We consider the system made of both the lake and its upstream region, and explore how to maintain the desirable ecological and economic properties of this system. In the viability framework, we translate these desirable properties into state constraints, then examine how, given the dynamics of the model and the available policy options, the properties can be kept. The set of states for which there exists a policy to keep the properties is called the viability kernel. We extend this framework to both major perturbations and long-term environmental changes. In our model, since the phosphorus inputs and outputs from the lake depend on rainfall, we will focus on extreme rainfall events and long-term changes in the rainfall regime. They can be described as changes in the state of the system, and may displace it outside the viability kernel. Its response can then be described using the concepts of resilience, vulnerability and adaptive capacity. Resilience is the capacity to recover by getting back to the viability kernel where the dynamics keep the system safe, and in this work we assume it to be the first objective of management. Computed for a given trajectory, vulnerability is a measure of the consequence of violating a property. We propose a family of functions from which cost functions and other vulnerability indicators can be derived for any trajectory. There can be several vulnerability functions, representing for instance social, economic or ecological vulnerability, and each representing the violation of the associated property, but these functions need to be ultimately aggregated as a single indicator. Due to the stochastic nature of the system, there is a range of possible trajectories. Statistics can be derived from the probability distribution of the vulnerability of the trajectories. Dynamic programming methods can then yield the policies which, among available policies, minimize a given trajectory. Thus, this viability framework gives indication on both the possible consequences of a hazard or an environmental change, and on the policies that can mitigate or avert it. It also enables to assess the benefits of extending the set of available policy options, and we define adaptive capacity as the reduction in a given vulnerability statistic due to the introduction of new policy options.