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Motion estimation on ocean satellite images by data assimilation in a wavelets reduced model

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The presentation concerns the issue of estimating surface motion from satellite images, either on rectangular windows or for a whole basin, with complex geographical boundary, such as Black Sea. The spatial domain is in all cases denoted by Ω.

The approach relies on a reduced model that is obtained by Galerkin projection of dynamic equations on subspaces of velocity and image fields. The dynamic model expresses usual assumptions such as the transport of image brightness by motion and advection-diffusion of velocity.

Subspaces of image and motion fields are obtained as finite dimension vectorial spaces with scalar and vector wavelets bases, respectiveley denoted ψ_l, for $l = 1 \dots L$ and ϕ_k, for $k = 1 \dots K$. A method has been defined in order to obtain the ψ_l and ϕ_k elements as solutions of constrained minimization problems described by:

$$\begin{cases} \min_{(\psi_1,\dots,\psi_n)\in\mathfrak{F}^n} \sum_{k=1}^{n} \mathcal{Q}(\psi_k) & \\ \mathcal{B}(\psi_k) = 0, & k = 1, ..., n \\ \langle \psi_j, \psi_k \rangle = \delta_{j,k} & \end{cases} \quad (1)$$

where:

- $\mathfrak{F}$ is a Hilbert functional space, with given inner product $\langle \cdot, \cdot \rangle$,
- $\mathcal{Q}(\psi) = \langle \mathcal{L}(\psi), \psi \rangle$ is a positively defined quadratic functional,
- $\mathcal{B}$ is a linear operator on $\mathfrak{F}$, used to express the chosen boundary conditions on image and motion fields,
- and $\delta_{j,k}$ is equal to 1 for $j = k$ and to 0 otherwise.

In order to define the scalar basis of the image subspace, the minimization problem is solved with $\mathcal{Q}(\psi) \triangleq \int_\Omega \nabla\psi(\mathbf{r})^2 \mathrm{d}^2\mathbf{r}$, and $\mathcal{B}(\psi) \triangleq \mathbf{n}(\mathbf{r})^\intercal \psi(\mathbf{r})$, where the vector $\mathbf{n}(\mathbf{r})$ is zero everywhere apart for the boundary $\partial\Omega$, and it is $\mathbf{n}(\mathbf{r}) = 1, \mathbf{n}(\mathbf{r}) \perp \partial\Omega, \forall \mathbf{r} \in \partial\Omega$. Thus, we are minimizing the smoothness $\int_\Omega \nabla\psi(\mathbf{r})^2 \mathrm{d}^2\mathbf{r}$ with the Neumann boundary conditions.

In order to define the vector basis of the motion subspace, the minimization problem (1), is considered with $\mathcal{Q}(\psi) \triangleq \int_\Omega \nabla\psi(\mathbf{r})^2 \mathrm{d}^2\mathbf{r}$, where $\psi : \Omega \mapsto \mathbb{R}^2$ is a planar vector field. As to the operator $\mathcal{B}(\psi)$, two possibilities are available:

$$\{\mathcal{B}(\psi)\}(\mathbf{r}) = \begin{cases} \mathbf{n}(\mathbf{r})^\intercal \psi(\mathbf{r}), & \mathbf{r} \in \partial\Omega \\ 0, & otherwise \end{cases}$$

$$\{\mathcal{B}(\psi)\}(\mathbf{r}) = \begin{cases} \mathbf{n}(\mathbf{r})^\intercal \psi(\mathbf{r}), & \mathbf{r} \in \partial\Omega \\ \{\mathrm{div}\psi\}(\mathbf{r}), & otherwise \end{cases}$$

In both cases, Neumann boudary conditions are applied, and in the second case, motion is considered as divergence-free.

The sequence of satellite images is projected on the image subspace in order to get a sequence of observation vectors: $b(t) = (\,b\,)_1\,(t) \dots b_L(t)^T$. These $b(t)$ are then assimilated in the reduced model in order to estimate the

values $a(t) = (a)_1(t) \ldots a_K(t)^T$, $a_i(t)$ being the coefficient of the projection of motion $\mathbf{w}(t)$ on the basis vector ψ. $\mathbf{w}(t)$ is then obtained as $\sum_{i=1}^{K} a_i(t)\psi_i$.

The approach has been used on satellite data acquired by NOAA-AVHRR sensors and on the MyOCean[1] analysis database for BlackSea.

[1]http://www.myocean.eu.org/