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On the nonlinear stage of modulation instability

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There is a point of common agreement that freak (or rogue) waves appear as a result of modulation instability of quasimonochromatic nonlinear waves. What is a nonlinear stage of modulation instability? In dimension $D = 2, 3$, the answer is known - modulation instability leads to formation of finite time singularities - collapses. In dimension $D = 1$ collapses are forbidden. However in this case development of modulation instability leads to formation of extreme waves where energy density exceeds the mean level in order of magnitude. As a result the study of long-time consequences of modulation instability is a problem of big practical importance, crucial for creation of a freak wave theory in ocean. In the recent time the simplest and most universal model for description of these waves is the focusing NLSE (Nonlinear Schrödinger Equation). In terms of the NLSE model we should study instability of the "condensate". The focusing NLSE is the model of the first approximation. For the surface of fluid this model describes the essentially weakly nonlinear quasimonochromatic wave trains. Nowadays a lot of models generalizing the NLSE are developed. Also, freak waves in the ocean were studied by numerical modeling of exact Euler equations for potential flow with free boundary. The behavior of freak waves studied by NLSE and by more sophisticated models shows considerable quantitative difference. Nevertheless, advanced improvement of NLSE does not lead to any qualitatively new effects. That means that a careful and detailed study of NLSE solutions is still very important problem. NLSE is a completely integrable system, having many exact solutions. It is natural to hope that the nonlinear development of the modulation instability is described by one of such solutions. We are interested only in instability growing from small spatially localized perturbation of condensate. Historically first such solution was found by Peregrine in 1983. Now "multi-Peregrine" are known. These solutions have a weak point - they are small perturbations of condensate only in the limit $t \to -\infty$. These solutions are homoclinic - they describe freak waves appearing "from nowhere" and completely disappearing in future. We find another class of exact solutions of NLSE which are small perturbation of condensate not at $t \to \infty$ but in the initial moment of time $t = 0$. This is the special class of $2N$-solitonic solutions of NLSE in presence of condensate. They grow exponentially, saturate and turn to a system of localized solitons propagating with fast group velocities in both directions. In general these solutions are not symmetric with respect to reflection of spatial coordinate. It is important that these solutions do not change a condensate phase. They describe the nonlinear stage of the modulation instability of a condensate.