Segregation induced fingering instabilities in granular avalanches

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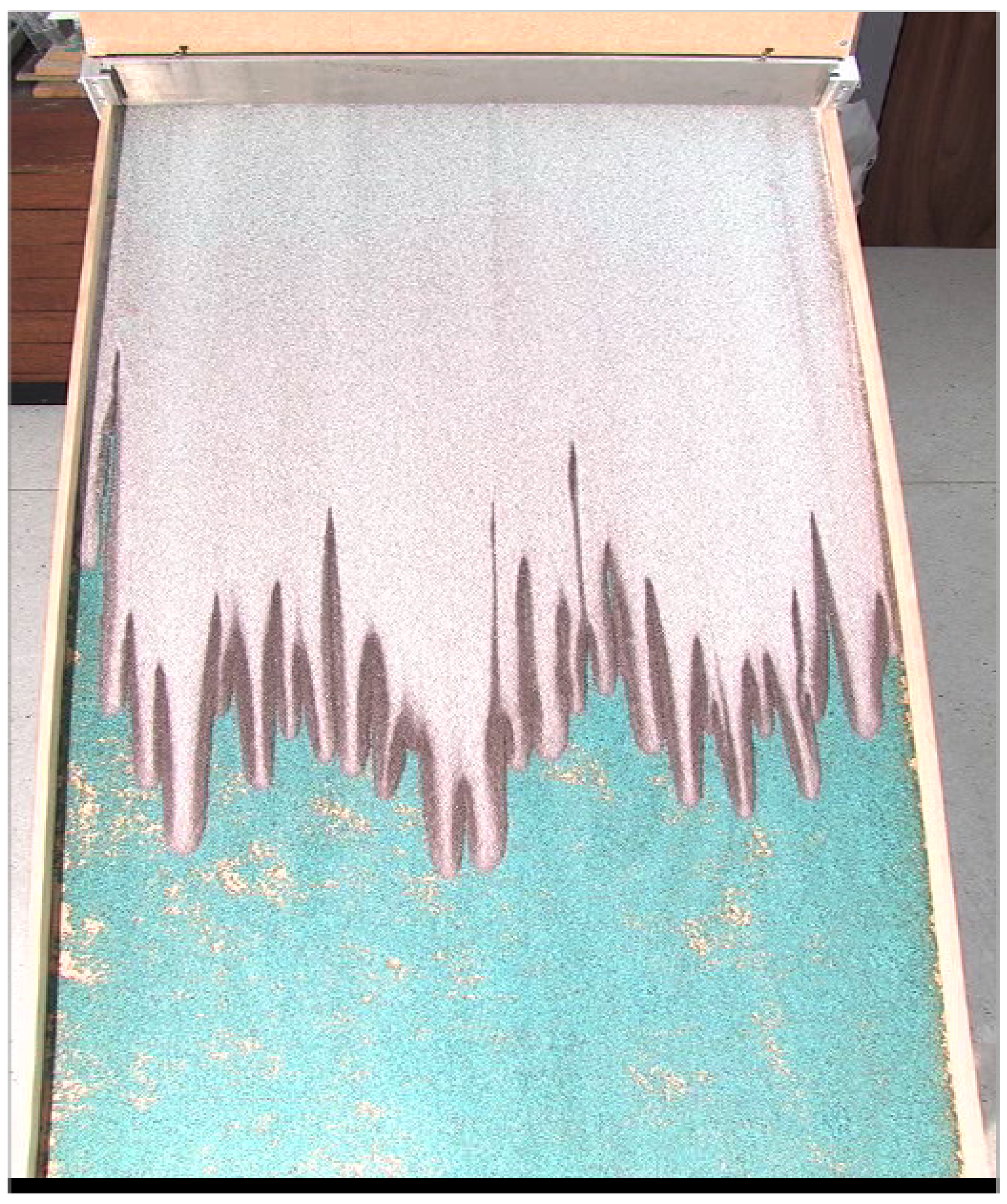
Introduction

It is known that particle size-segregation can lead to the transport of large particle to the flow margins and form bouldery flow fronts. In many natural flows these bouldery margins experience a much greater frictional force. This segregation-mobility feedback often causes frontal instabilities (see picture below). Here, we present a continuum model designed to capture this effect.

Pumiceous lobes and levees. Main picture: Aerial view of 7 August 1980 pumice-rich lobes and levees on the Mount St. Helens ignimbrite fan.

Experiments



Bidisperse mixture of spherical (white) glass ballotini (75-150 microns) and irregular (brown) carborundum grains (315-350 microns)

Conclusions

- **This is the first model of segregation-mobility feedback.**
- **Captures the break-up of a uniform front.**
- **Model linearly unstable to arbitrarily small perturbations.**
- **Could be stabilised by including additional physical effects.**

Reference

[1] Segregation-induced fingering instabilities in granular free-surface flows. M. J. Woodhouse A. R. Thornton, C. G. Johnson, B. P. Kokelaar and J. M. N. T. Gray. **J. Fluid Mech. (2012), vol. 709 pp 543-580**

Model, published in [1]

Assumptions

- Based on traditional Savage-Hutter shallow layer model.
- Depth averaged size segregation model used for the evolving particle distribution.
- Coupled via a friction that evolves with the size distribution.

$$\frac{\partial h}{\partial t} + \frac{\partial}{\partial x}(hu) + \frac{\partial}{\partial y}(hv) = 0,$$

$$\frac{\partial}{\partial t}(hu) + \frac{\partial}{\partial x}(hu^2) + \frac{\partial}{\partial y}(huv) + \frac{1}{2}\frac{\partial}{\partial x}(gh^2\cos\theta) = gh\left(\sin\theta - \mu\frac{u}{\sqrt{u^2+v^2}}\cos\theta\right)$$

$$\frac{\partial}{\partial t}(hv) + \frac{\partial}{\partial x}(huv) + \frac{\partial}{\partial y}(hv^2) + \frac{1}{2}\frac{\partial}{\partial y}(gh^2\cos\theta) = hg\left(-\mu\frac{v}{\sqrt{u^2+v^2}}\cos\theta\right)$$

$$\frac{\partial}{\partial t}(hC) + \frac{\partial}{\partial x}(huC) + \frac{\partial}{\partial y}(hvC) = (1-\alpha)\left(\frac{\partial}{\partial x}(huC(1-C)) + \frac{\partial}{\partial y}(h\,vC(1-C))\right).$$

where

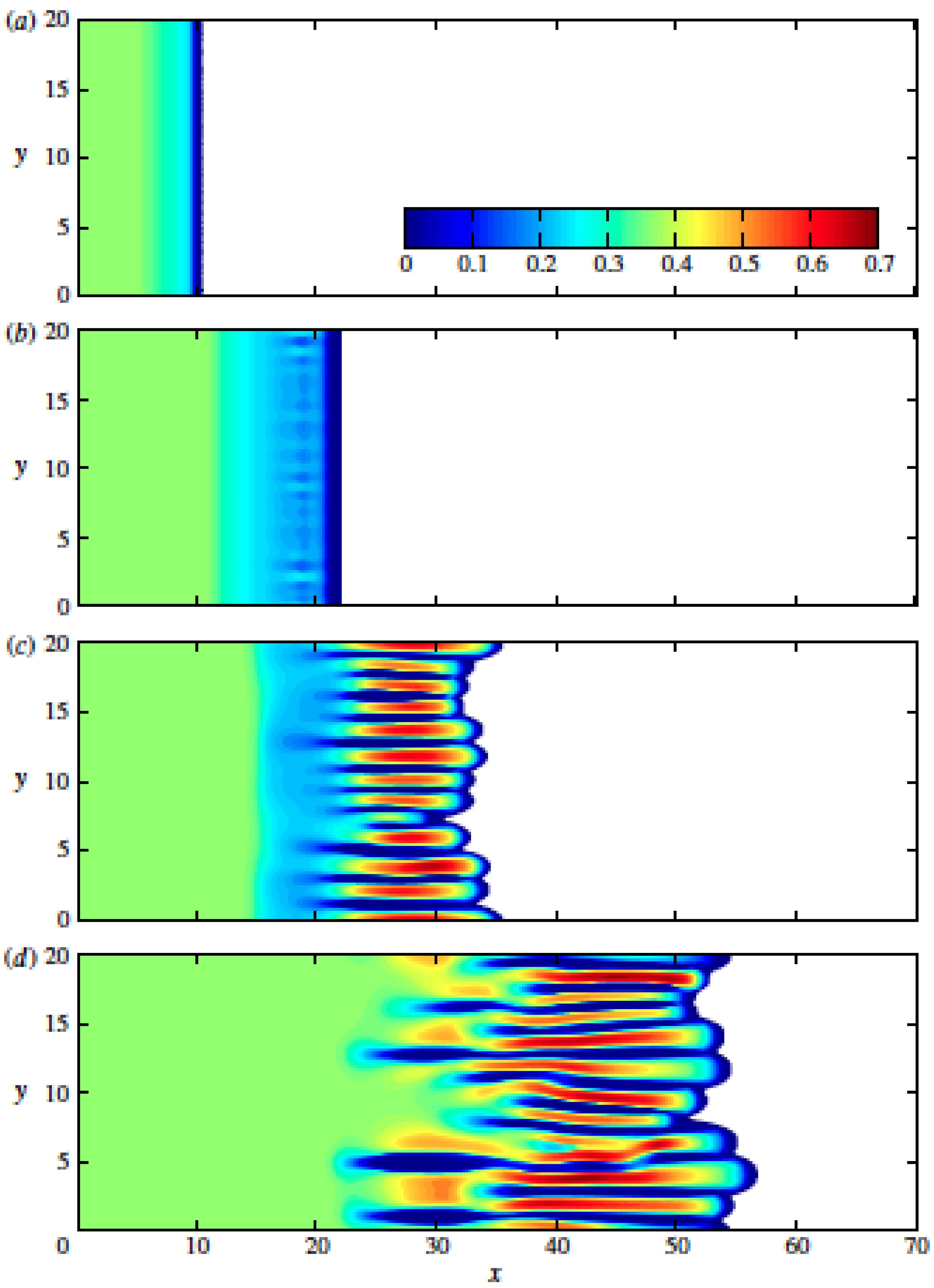
$$\mu = C\mu^s + (1-C)\mu^l$$

and

$$\mu^\nu(h,u) = \tan\delta_1^\nu + [\tan\delta_2^\nu - \tan\delta_1^\nu]\exp\left\{\frac{-\sqrt{g}\beta h^{3/2}}{L^\nu u}\right\}$$

h	Depth of the flow	u	Down-slope velocity
v	Cross-slope velocity	C	Concentration of small particles

Numerical Solution, published in [1]



Contour of the velocity for time (a) t=30; (b) t=78; (c) t=120 and (d) t=195 from the solution of the model.