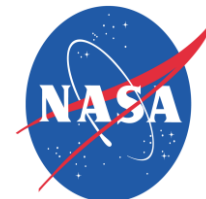




Parametrization of the raindrop size distribution

E. Adirosi ^{1,2}, A. Tokay ^{3,4}, L. Baldini ¹ and N. Roberto ¹

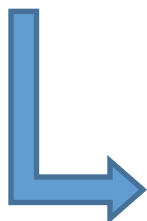
- 1 Institute of Atmospheric Sciences and Climate, CNR, Rome, Italy
- 2 University of Rome Sapienza, Rome, Italy
- 3 Joint Center for Earth Systems Technology, UMBC, Baltimore, Maryland
- 4 NASA-GSFC, Greenbelt, Maryland



MOTIVATIONS

Knowledge of the drop size distribution (DSD) is essential in the retrieval of precipitation from remote sensing measurements

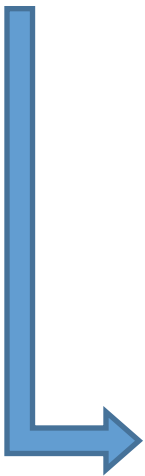
- Precipitation retrieval algorithm from the NASA/JAXA Tropical Rainfall Measuring Mission (TRMM) precipitation radar (PR) employs a three-parameter gamma DSD with a fix shape parameter ($\mu = 3$).
- NASA/JAXA Global Precipitation Measurement (GPM) will retrieve parametrization of gamma distributions from dual-frequency precipitation radar (DPR) measurements



- Fixing one of the three parameters of the gamma distribution
- Using an analytical relationship between two of the parameters

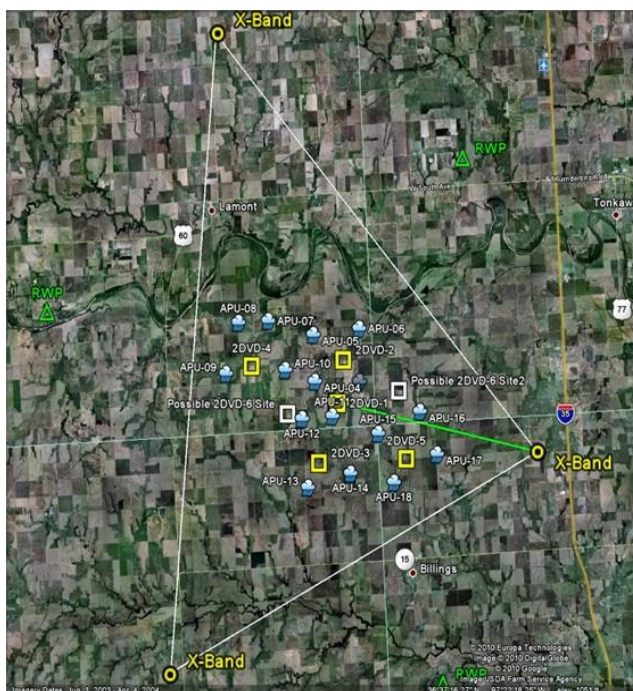
OBJECTIVES

To investigate the parameters of gamma DSD using disdrometer measurements

- 
- A large, thick blue arrow points downwards from the main objective box to the first step of the list.
- 1 Derive relationships between integral rainfall and DSD parameters as a function of Ku- and Ka-band reflectivity
 - 2 Investigate the performances of different gamma DSD fitting methods
 - 3 Apply various fitting methods to the relations between gamma DSD parameters
 - 4 Determine the role of the truncation on the integral rainfall parameters

SITE AND INSTRUMENTATION

- In this study the two-dimensional video disdrometer (2DVD) observations from Midlatitude Continental Convective Clouds Experiment (MC3E) have been used.



<http://gpm.nsstc.nasa.gov/mc3e/>

Distances between 2DVDs	
min	0.35 km
max	9.2 km

MC3E

- Field campaign of the GPM Ground Validation program
- Collaborative effort between DOE and NASA
- North Central Oklahoma
- April – June 2011
- Seven two dimensional video disdrometers

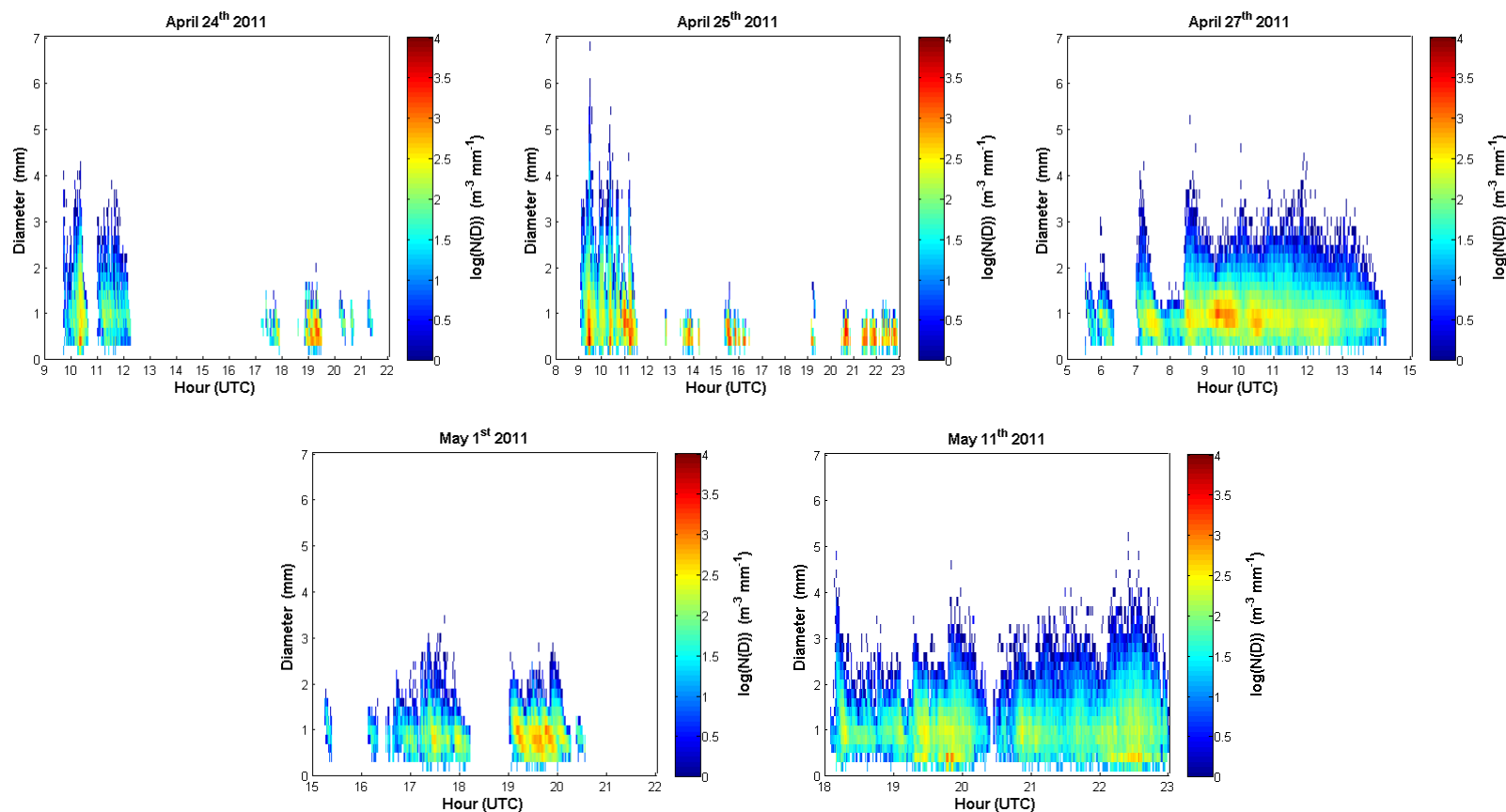
2DVD RAW DATA

- Diameter (mm)
- Fall velocity (m s^{-1})
- Oblateness (dimensionless)



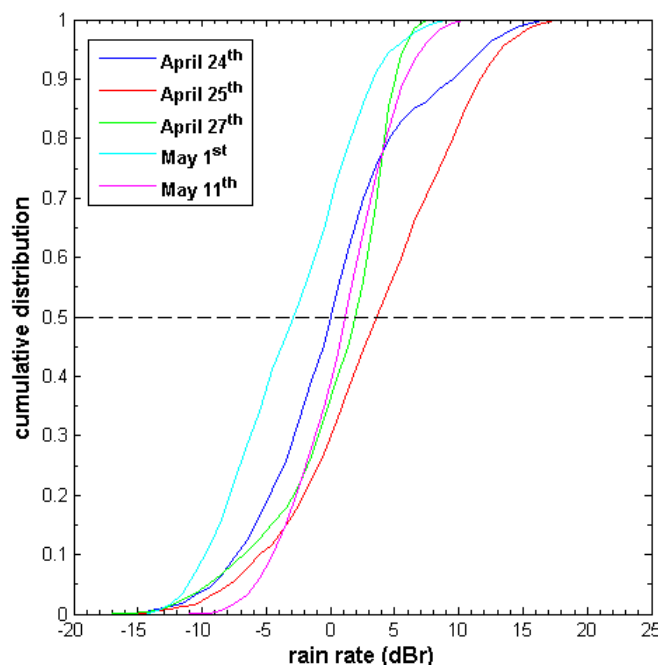
DATA ANALYSIS

- The drops outside the $\pm 50\%$ of Gunn and Kinzer observations were removed and the rain vs. no-rain thresholds were taken to be 50 drops and $D_{\max} \geq 1$ mm. From seven disdrometers we obtained 9,985 1-min DSDs.



DATA ANALYSIS

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Day	Mean(R)	Mean Rain Minutes	D_{\max}	R_{cum}
	mm h ⁻¹		mm	mm
April 24 th	3.45	155	7.43	62.6
April 25 th	6.15	152	8.07	109.5
April 27 th	1.81	394	5.27	84.8
May 1 st	1.02	84	4.31	10.1
May 11 th	1.99	227	7.56	53.0

April 25th → very heavy rain

May 1st → light rain

Maximum rain drop diameter measured during MC3E field campaign → 9.5 mm

FITTING METHODS

The three-parameter gamma distribution is expressed as

$$N(D) = N_0 D^\mu \exp(\Lambda D)$$

- ❑ **COMPLETE GAMMA FUNCTION** → the integration over particle size is extended from zero to infinity.
- ❑ **INCOMPLETE GAMMA FUNCTION** → accounts for the truncation effects due to the fact that the observed DSDs are bounded by minimum and maximum diameters.
- ❑ It is also feasible that the observed DSD can not be well represented even by the incomplete gamma function.

Fitting methods for the computation of the DSD parameters:

- 1) **D_0 - D_{mass} Method**
- 2) **σ_{mass} - D_{mass} Method**
- 3) **Method of Moments (MM)**
- 4) **Integral Parameters Minimization Method (IPMM)**
- 5) **DSD Minimization Method (DMM)**

FITTING METHODS

1) D_0 - D_{mass} Method

$$N_0 \int_{D_{min}}^{D_0} D^{3+\mu} e^{-\Lambda D} dD = N_0 \int_{D_0}^{D_{max}} D^{3+\mu} e^{-\Lambda D} dD$$

$$D_{mass} = \frac{N_0 \int_{D_{min}}^{D_{max}} D^{4+\mu} e^{-\Lambda D} dD}{N_0 \int_{D_{min}}^{D_{max}} D^{3+\mu} e^{-\Lambda D} dD}$$

μ and Λ for complete and incomplete gamma function

2) σ_{mass} - D_{mass} Method

$$\sigma_{mass} = \frac{N_0 \int_{D_{min}}^{D_{max}} (D - D_{mass})^2 D^{3+\mu} e^{-\Lambda D} dD}{N_0 \int_{D_{min}}^{D_{max}} D^{3+\mu} e^{-\Lambda D} dD}$$

$$D_{mass} = \frac{N_0 \int_{D_{min}}^{D_{max}} D^{4+\mu} e^{-\Lambda D} dD}{N_0 \int_{D_{min}}^{D_{max}} D^{3+\mu} e^{-\Lambda D} dD}$$

μ and Λ for complete and incomplete gamma function

3) Method of Moments

$$M_P = N_0 \int_{D_{min}}^{D_{max}} D^{P+\mu} e^{-\Lambda D} dD$$

(MM012, MM246, MM234, MM346, MM456)

N_0 μ and Λ for complete and incomplete gamma function

FITTING METHODS

4) Integral Parameters Minimization Method

The retrieval can be done through minimizing the integral parameter (e.g. rain rate or reflectivity) that is directly calculated from disdrometer observations and from normalized gamma distribution

$$N(D) = \frac{N_t}{D_{mass}} \frac{(4+\mu)^{\mu+1}}{\Gamma(\mu+1)} \left(\frac{D}{D_{mass}} \right)^{\mu} \exp \left[-(4+\mu) \frac{D}{D_{mass}} \right]$$

$$N(D) = N_w \frac{6}{256} \frac{(4+\mu)^{\mu+4}}{\Gamma(\mu+4)} \left(\frac{D}{D_{mass}} \right)^{\mu} \exp \left[-(4+\mu) \frac{D}{D_{mass}} \right]$$

R_{gamma} or Z_{gamma}

μ for complete gamma function

5) DSD Minimization Method

The shape parameter can be retrieved through minimizing the absolute deviation between the normalized observed DSD and the normalized gamma distribution (as in Bringi et al., 2003, J. Atmos. Sci., 60, 354-365).

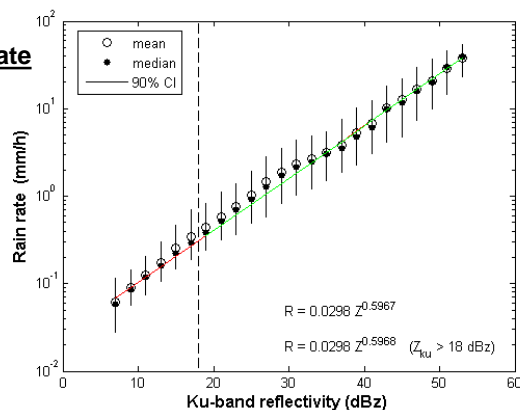
RESULTS

1

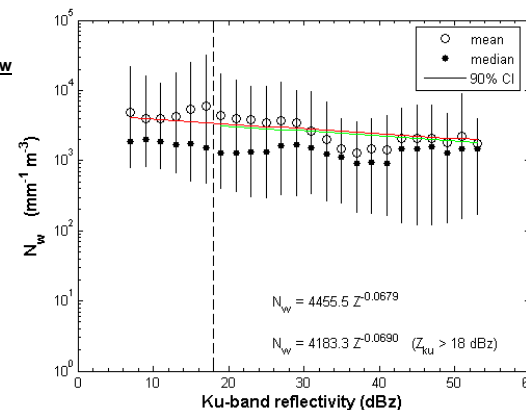
Relationships between rainfall and DSD parameters as a function of Ku-band reflectivity



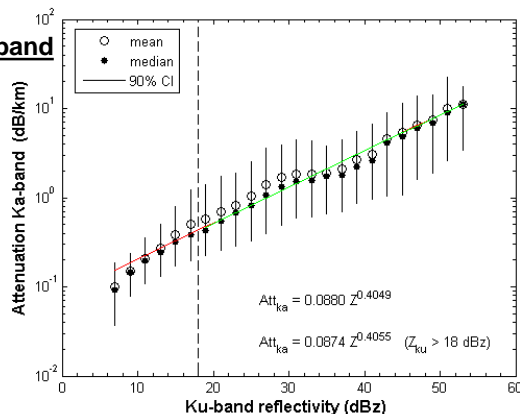
Rain Rate



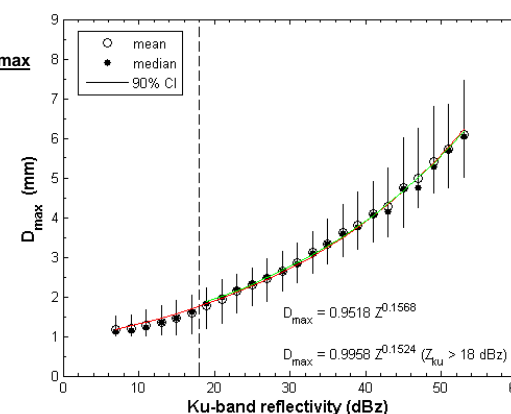
N_w



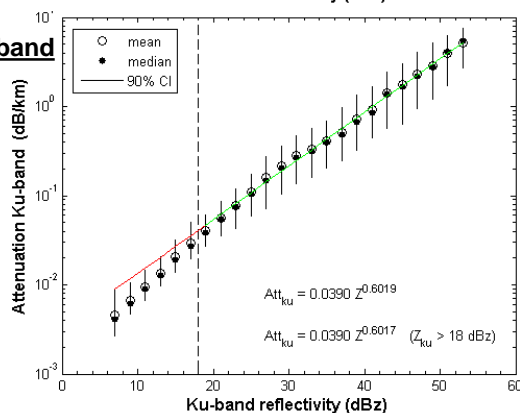
Attenuation K_a -band



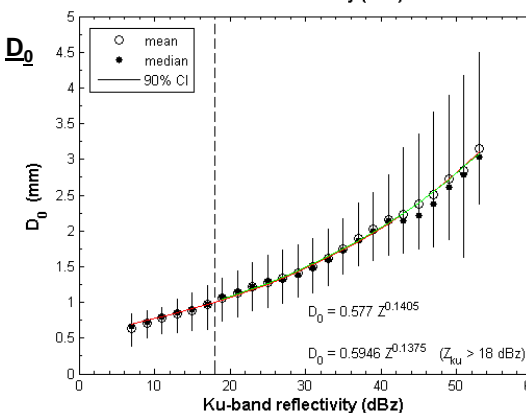
D_{max}



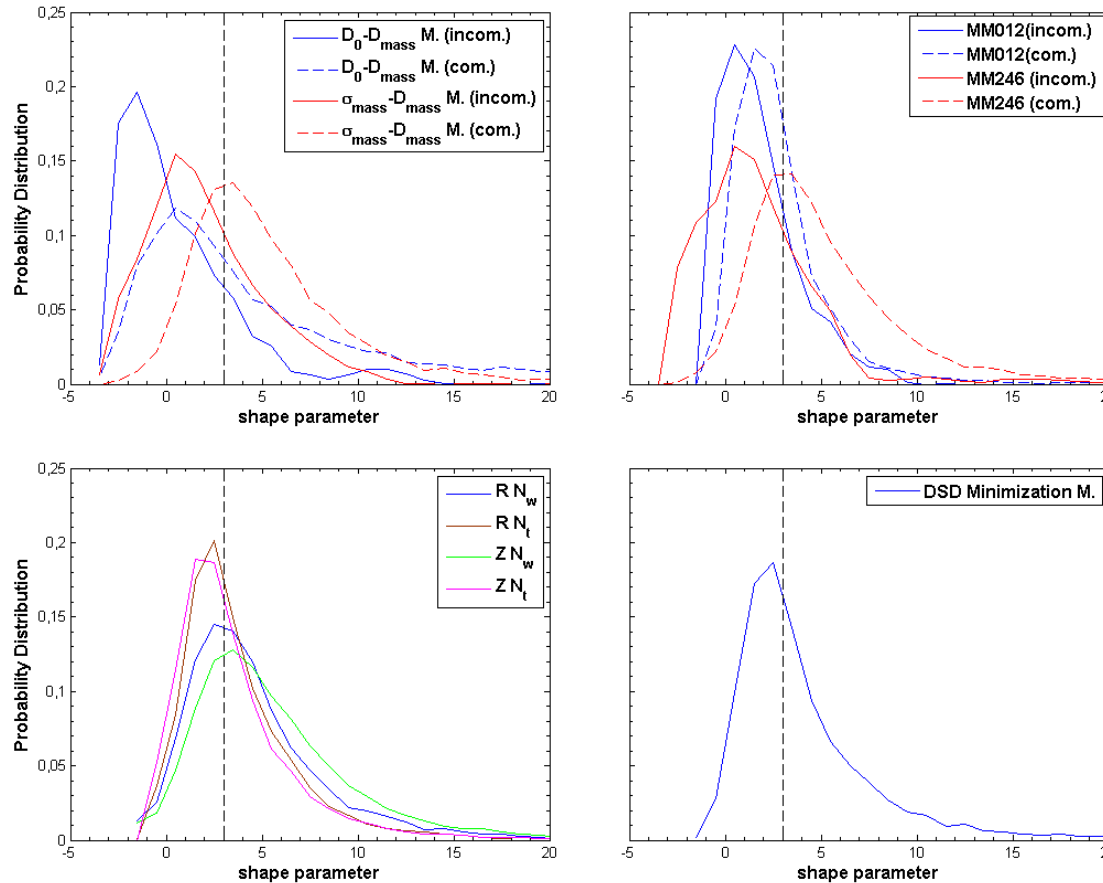
Attenuation K_u -band



D_0



Shape Parameter



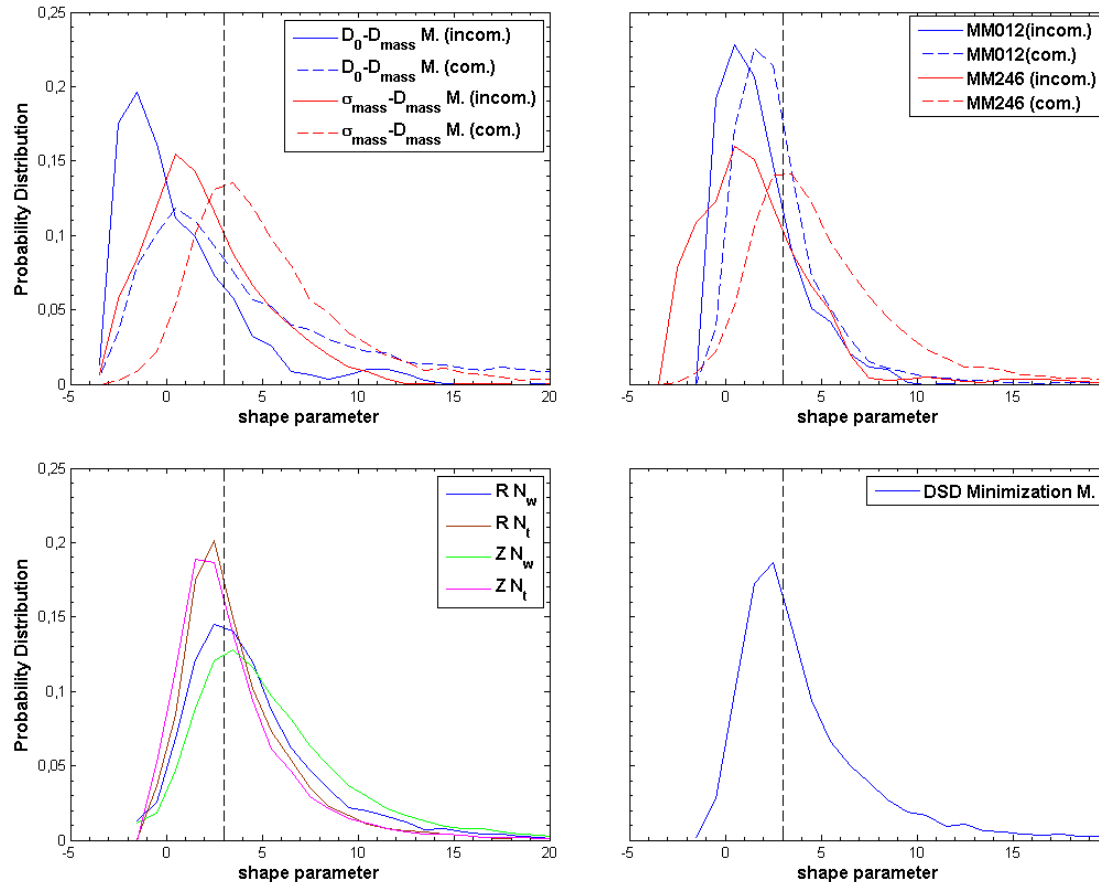
- The distributions are narrower and the means shift to the left when truncation effects are considered.

- The distributions for complete and incomplete gamma are distinctly different for higher MM (e.g. the medians are 5.7 and 0 when MM456 is used).

	D_0 - D_{mass} M.		σ_{mass} - D_{mass} M.		MM012		MM246		IPMM (N_t^*)		IPMM (N_w)		DMM
	Com	Incom	Com	Incom	Com	Incom	Com	Incom	R	Z	R	Z	
mean	3.94	0.56	5.12	2.00	2.77	1.78	5.00	1.65	3.77	3.50	4.58	4.45	4.02
median	2.43	-0.3	4.34	1.52	2.26	1.37	4.21	1.20	3.00	2.80	3.80	4.60	3.10



Shape Parameter



Range of variability
 $-4 < \mu < 20$

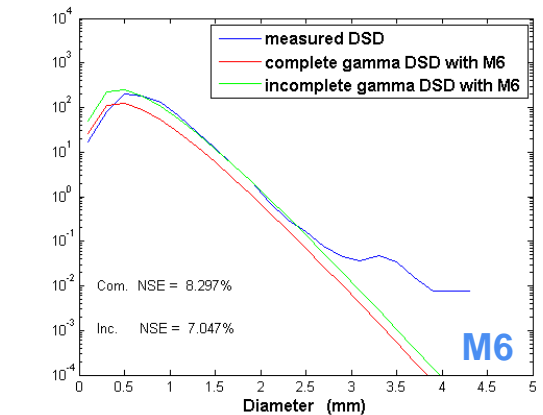
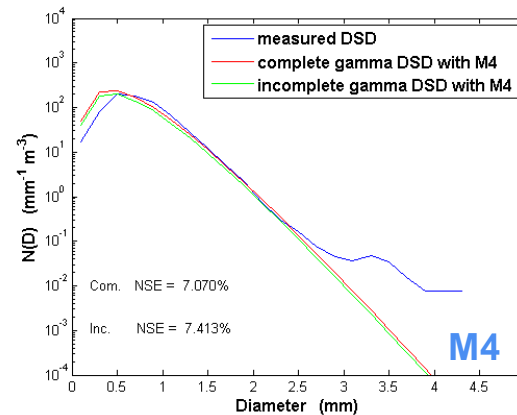
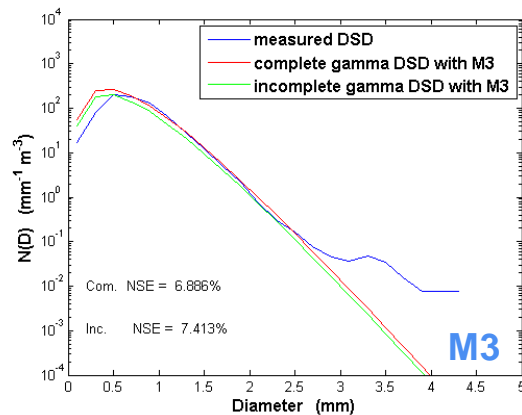
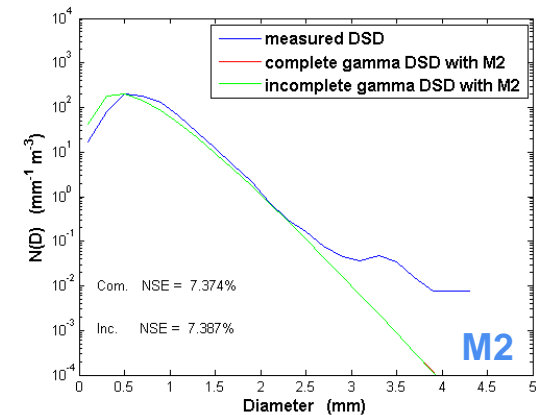
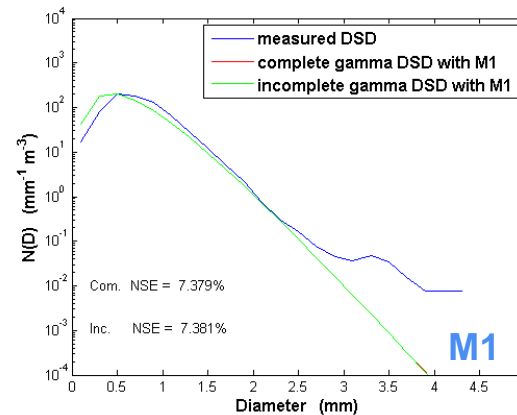
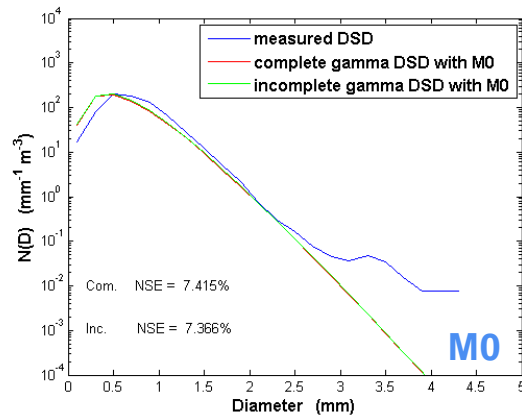
- The high percentages of the D_0 - D_{mass} Method is most due to the fact that $D_0 > D_{\text{mass}}$ in 23% of the time.

- Usually if truncation effects are considered the percentage of observations outside the range decreases.

	D_0 - D_{mass} M.		σ_{mass} - D_{mass} M.		MM012		MM246		IPMM (N_t^*)		IPMM (N_w)		DMM
	Com	Incom	Com	Incom	Com	Incom	Com	Incom	R	Z	R	Z	
% Out	35.22	12.19	2.19	0.80	1.26	5.17	1.85	1.14	0.75	1.03	3.32	2.44	2.14

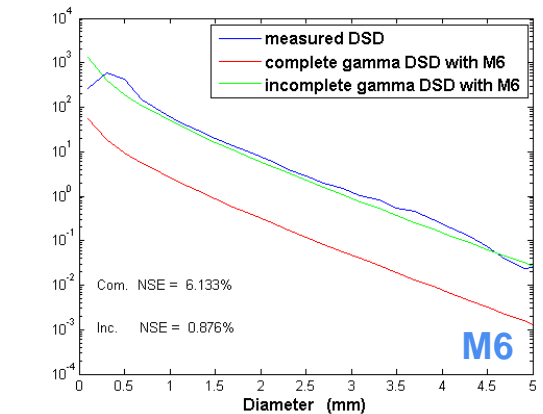
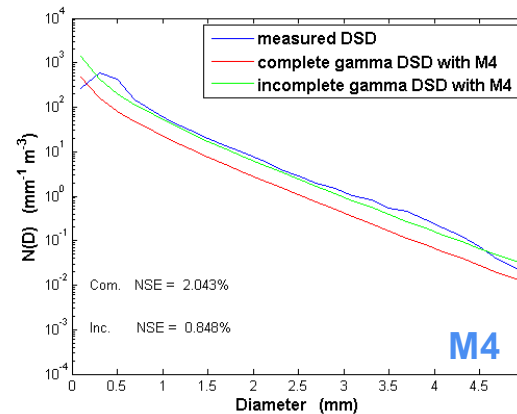
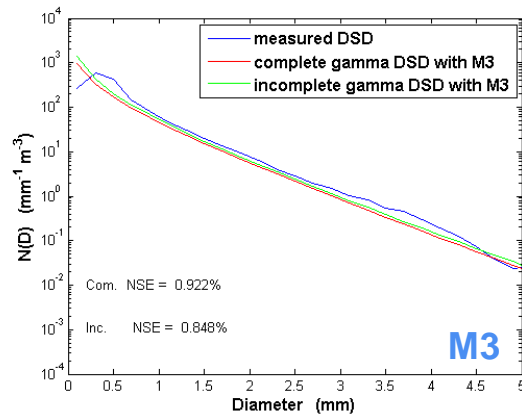
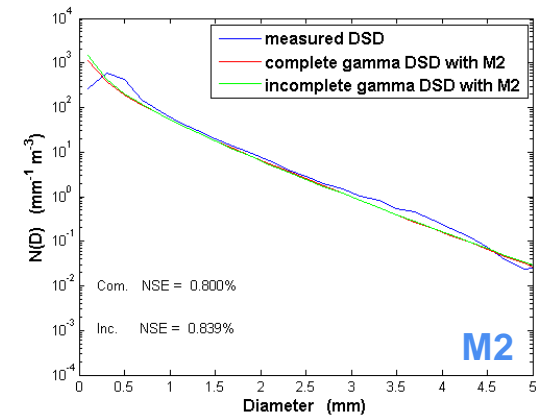
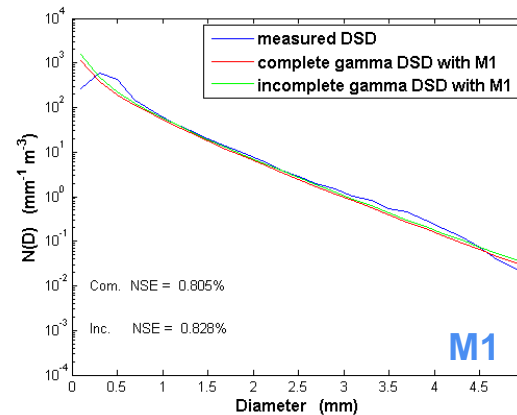
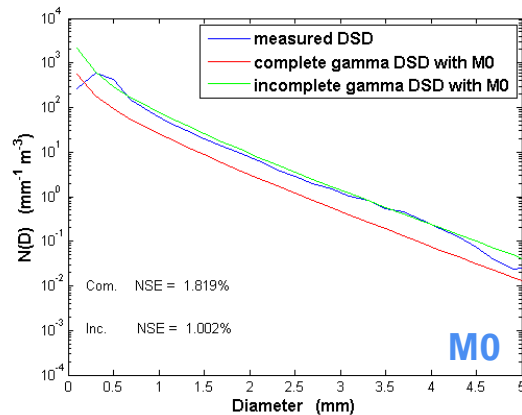
$D_0 - D_{\text{mass}}$ Method (May 1st → light rain)

Normalised standard error $\rightarrow NSE = \frac{\sqrt{(N(D)_{\text{meas.}} - N(D)_{\text{gamma}})^2}}{N(D)_{\text{meas.}}}$



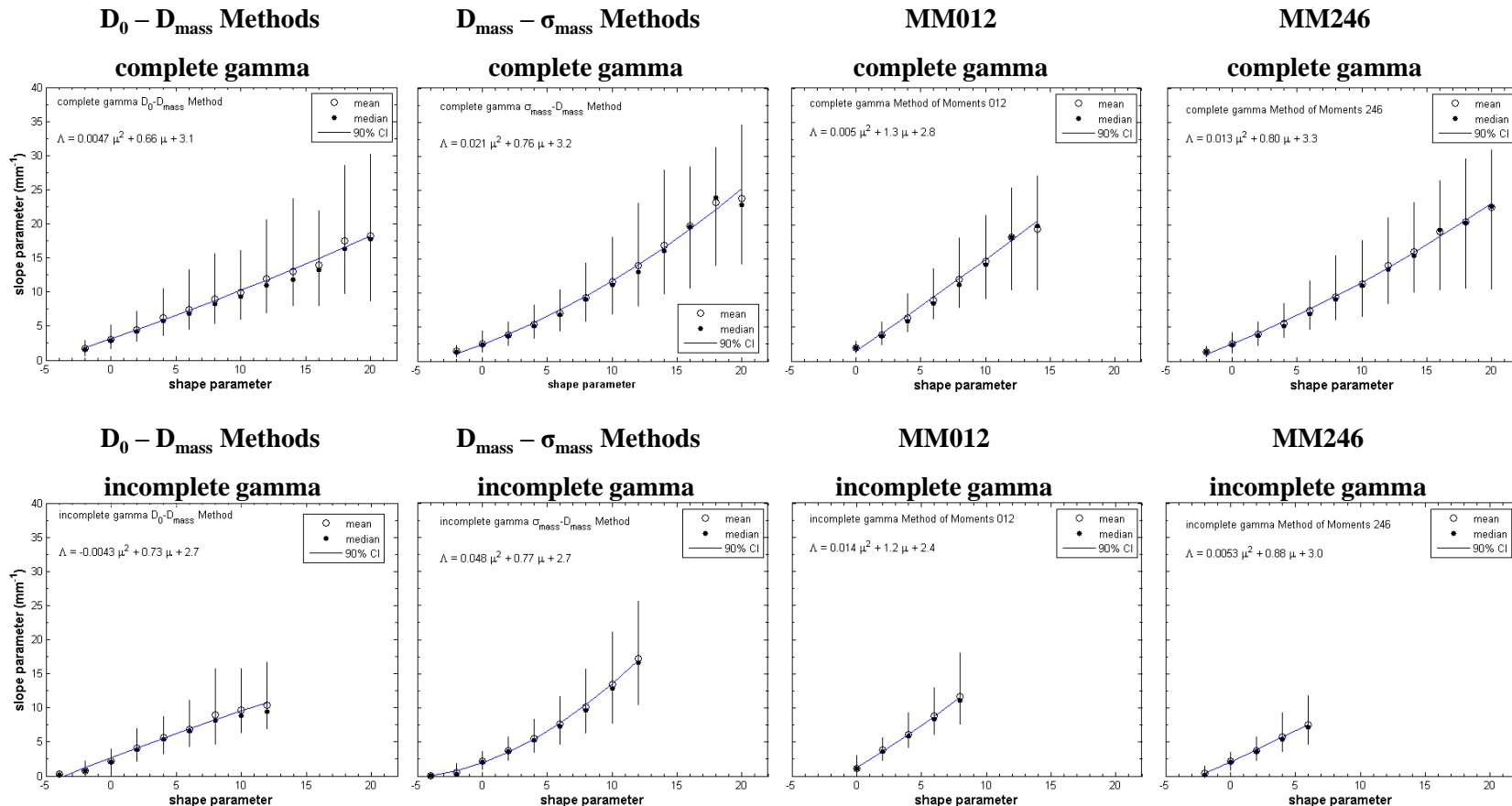
$D_0 - D_{\text{mass}}$ Method (April 25th → very heavy rain)

Normalised standard error →
$$NSE = \frac{\sqrt{(N(D)_{\text{meas.}} - N(D)_{\text{gamma}})^2}}{N(D)_{\text{meas.}}}$$



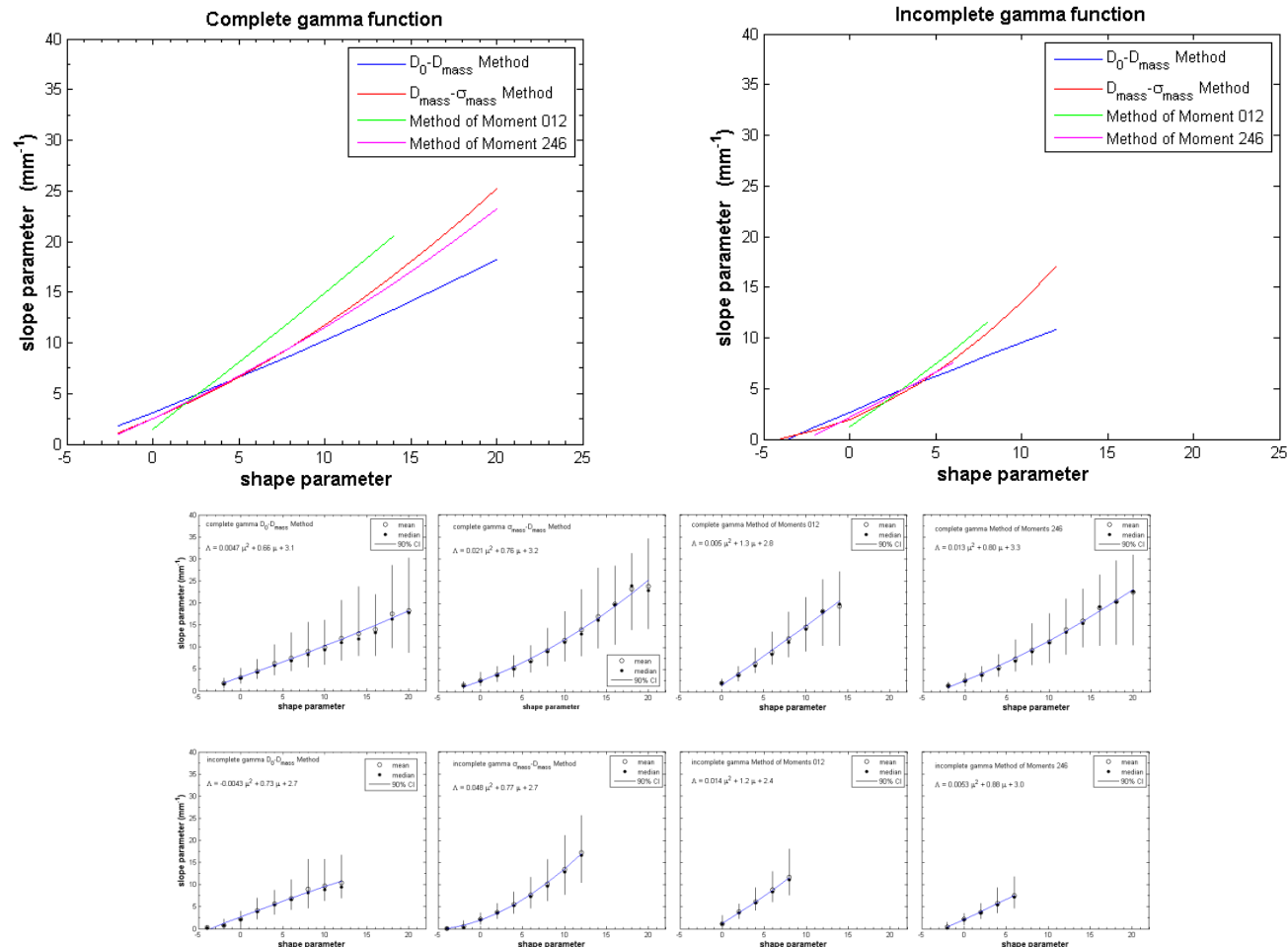
$\mu - \Lambda$ Relationship

- A relation between two parameters of gamma DSD can be considered as an alternative to a constant shape parameter in the GPM dual-frequency precipitation radar algorithm.



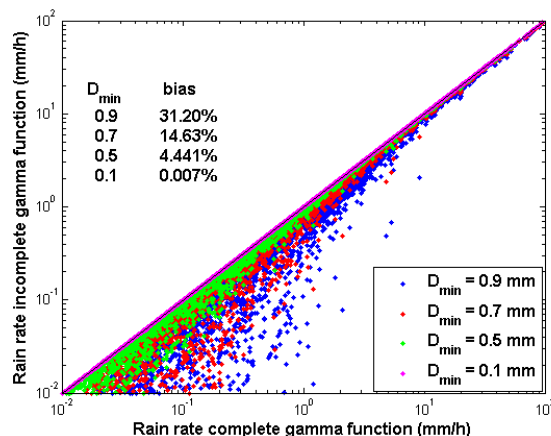
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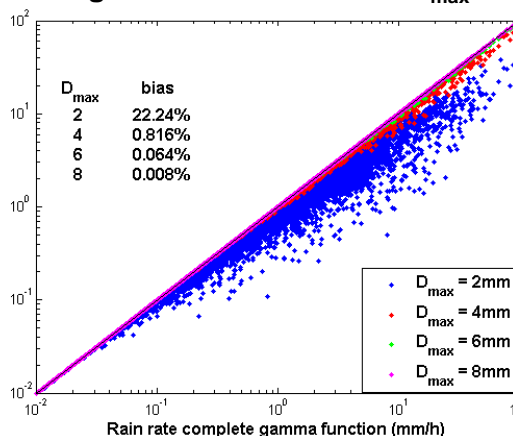


Rain Rate

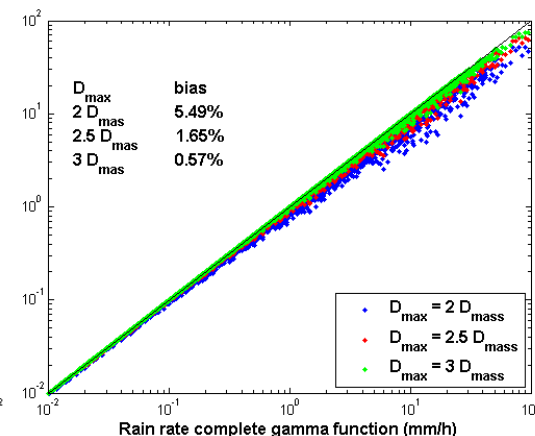
Left Truncation



Right Truncation with fix D_{max}



Right Truncation with $D_{max} = f(D_{mass})$

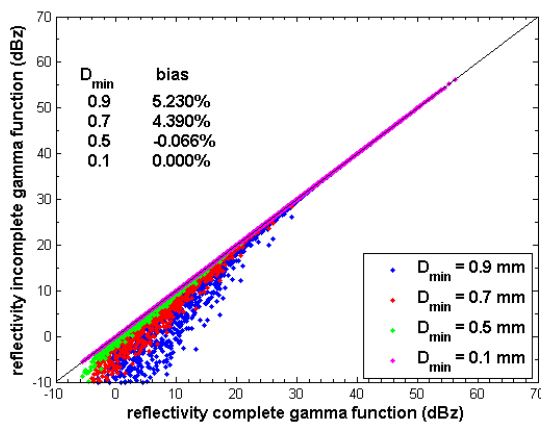


Bias < 1% $\rightarrow D_{min} \leq 0.3$ mm

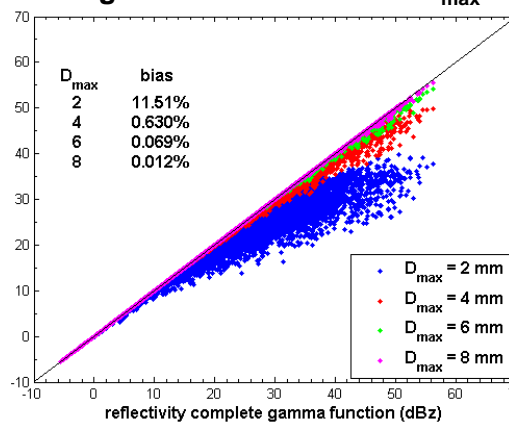
Bias < 1% $\rightarrow D_{max} \geq 4$ mm

Reflektivitiy

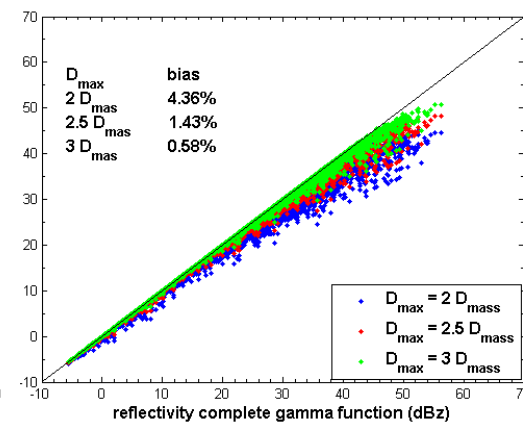
Left Truncation



Right Truncation with fix D_{max}



Right Truncation with $D_{max} = f(D_{mass})$



Bias < 1% $\rightarrow D_{min} \leq 0.9$ mm

Bias < 1% $\rightarrow D_{max} \geq 4$ mm

FUTURE WORK

- Processing two dimensional video distrometer observations from different field campaigns

LPVEx (Finland, 2010)

HyMex SOP1 (Southern France and Central Italy, 2012)

Iowa Floods (Iowa, 2013)

HMT-SE (Carolina, 2014)

NASA Wallops Flight Facility

- Fitting the complete and incomplete gamma function to the observed DSDs using the different methods presented earlier.



Thank you!!