



## **An Efficient Q2P1 Finite Element Discretisation and Preconditioner for Variable Viscosity Stokes Systems**

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Here I describe a numerical method suitable for studying 3D non-linear, large deformation processes associated with crustal and lithospheric deformation. The method employs a combination of mixed finite elements for the flow problem, coupled to the Material-Point-Method for representing material state and history variables.

This computational methodology is intended to simultaneously satisfy all of the geodynamic modelling requirements. Particular emphasis is given to the development of non-linear solvers and preconditioners which are performant, practical and highly scalable – thereby enabling high resolution 3D simulations to be performed using massively parallel computational hardware.

We have made a number of fundamental design choices which result in a fast, highly scalable and robust Q2P1 finite element implementation which is suitable for solving a wide range of geodynamic applications. Specifically these choices include: (i) utilizing an inf-sup stable mixed finite element (with a mapped pressure space) which provides a reliable velocity and pressure solution; (ii) expressing the problem in defect correction form so that Newton-like methods can be exploited; (iii) making extensive use of matrix-free operators which both drastically reduces the memory requirements and improves the parallel scalability of the sparse matrix-vector product; (iv) deferring a wide range of choices associated with the solver configuration to run-time.

The performance characteristics of our hybrid geometric multi-grid preconditioning strategy is presented. The robustness of the preconditioner with respect to the viscosity contrast and the topology of the viscosity field, together with the parallel scalability is demonstrated. We will highlight the benefits of using hybrid coarse grid hierarchies consisting of a combination of Galerkin, assembled and matrix-free operators. The merits of using aggressive coarsening strategies will also be discussed. Examples from 3D continental rifting and visco-plastic folding experiments are presented to demonstrate the efficiency of the new methodology.