



A discontinuous Galerkin numerical model for the simulation of multiphase gas-particle flows in explosive volcanic eruptions

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During explosive volcanic eruptions a mixture of gases, magma fragments, crystals and eroded rocks is injected in the atmosphere at high velocity, pressure and temperature. In the proximity of the volcanic vent, the erupted underexpanded multiphase mixture can manifest the features of supersonic flows, while the subsequent column behaviour is controlled by the (subsonic) turbulent mixing and mass and thermal exchange between the gas-particle mixture and the atmosphere.

One of the main difficulties of the numerical simulation of explosive volcanic eruptions is therefore the need of modeling a multiphase process where different fluid dynamic regimes coexist and develop on a wide range of temporal and spatial scales.

From a computational point of view, this requires robust numerical techniques able to resolve supersonic regimes and to capture flow discontinuities (shock waves), as well as to reduce, where needed, the so-called numerical diffusion (while increasing the numerical accuracy) in order to simulate gas-particle non-equilibrium phenomena. Several examples of numerical approximation of multiphase gas-particle equations based on finite volume approach have been proposed in the literature, able to simulate the multiphase mixture up to second-order accuracy in space and time. However, achieving higher order of accuracy in the finite volume framework implies an increasing computational cost related to the extension of the computational stencil, in particular when a parallel implementation has to be employed.

In this work, a mixture of gas and solid particles is described with a set of coupled partial differential equations for the mass, momentum and energy of each phase. Solid particles and the gas phase are considered as non-equilibrium interpenetrating continua, following an Eulerian-Eulerian approach. Each phase is compressible and inviscid. The gas and particles dynamics are coupled through the drag term in the momentum equations and the heat exchange term in the energy equations.

As an alternative to existing finite volume methods, a p-adaptive discontinuous Galerkin space discretization is proposed for the multiphase conservation equations, following the method of lines. An operator splitting approach is adopted, coupled with explicit Runge-Kutta methods for advective terms and semi-implicit time averaging methods for interphase exchange terms.

Discontinuous Galerkin methods can be interpreted as an extension of finite volume methods to arbitrary order of accuracy. As finite volume methods, discontinuous Galerkin methods provide discrete conservation laws that reproduce at the discrete level the fundamental physical balances characterizing the continuous problem. Moreover, both the methods represent a good choice for the approximation of problems whose solution presents discontinuities, i.e. explosive volcanic phenomena. However, with discontinuous Galerkin methods high order accuracy can be obtained without extending the computational stencil, thus allowing for a good scalability on parallel architectures.

Limiting techniques are introduced on the advective terms of the system, in order to prevent the formation of spurious oscillations and avoid unphysical negative values in the numerical solution. An automatic criterion is introduced to adapt the local number of degrees of freedom and to improve the accuracy locally. The employed technique is simple and relies on the use of orthogonal hierarchical basis functions. The p-adaptivity algorithm allows to reduce the computational cost while maintaining the accuracy of the numerical approximation.

The numerical model is applied and tested to several relevant test cases, with special focus on pyroclastic flows arising in volcanic eruptions, in order to assess its accuracy and stability properties. Moreover we analyse the efficiency of the p-adaptivity approach.