



## Waves in geomaterials with a negative modulus

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The presence of descending branches of loading curves is a well-known feature of geomaterials. It is non-reversible and usually associated with different types of load-induced damage accumulation. There however exist mechanisms that produce reversible descending branch giving rise to negative elastic stiffness (modulus). One such mechanism is rotation of non-spherical grains under shear stress in the presence of high compressive load.

The presence of negative moduli breaks positive definiteness of the elastic energy, such that the material with negative moduli is unstable and hence cannot exist on its own. It can however be statically stable as a part of an encompassing system that is stiff enough to stabilise the negative stiffness material. In this case it is important to consider its dynamic stability, in particular the possibility of wave propagation in such a material.

Geomaterials with rotating particles require the introduction of a Cosserat continuum for their modelling. We concentrate on the simplest case of isotropic Cosserat continuum, governed by six independent moduli: two conventional Lamé constants and four Cosserat moduli controlling the relationship between the stress and moment stress on the one hand and the internal rotations and their gradients on the other. The (infinitesimal) rotation of non-spherical particles can make negative the Cosserat modulus that relates the internal (particle) rotations with non-symmetric parts of stress tensor.

We demonstrate that there exists a range of values of this negative modulus at which the wave propagation is still possible such that all four types of waves (p-wave, two shear waves and a twist wave) exist. We call the values of the modulus in this range, the admissible values of negative modulus. Furthermore, while in the conventional isotropic Cosserat continuum the twist wave and one of the shear waves exist only at high frequencies, higher than a certain threshold frequency, the presence of the negative modulus removes this threshold. Therefore the negative modulus of admissible values makes all four waves exist in all frequencies. Thus the observations (measurements) of the twist wave and the second shear wave can potentially provide a method of detecting the presence of the negative modulus and characterising the mechanism of its generation.

Keywords: Rolling of non-spherical particles, Cosserat continuum, Twist waves, Frequency threshold