



Multigrid solvers and multigrid preconditioners for the solution of variational data assimilation problems

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In order to lower the computational cost of the variational data assimilation process, we investigate the use of multigrid methods to solve the associated optimal control system. On a linear advection equation, we study the impact of the regularization term on the optimal control and the impact of discretization errors on the efficiency of the coarse grid correction step. We show that even if the optimal control problem leads to the solution of an elliptic system, numerical errors introduced by the discretization can alter the success of the multigrid methods. The view of the multigrid iteration as a preconditioner for a Krylov optimization method leads to a more robust algorithm. A scale dependent weighting of the multigrid preconditioner and the usual background error covariance matrix based preconditioner is proposed and brings significant improvements.

[1] Laurent Debreu, Emilie Neveu, Ehouarn Simon, François-Xavier Le Dimet and Arthur Vidard, 2014: Multigrid solvers and multigrid preconditioners for the solution of variational data assimilation problems, submitted to QJRMS, <http://hal.inria.fr/hal-00874643>

[2] Emilie Neveu, Laurent Debreu and François-Xavier Le Dimet, 2011: Multigrid methods and data assimilation - Convergence study and first experiments on non-linear equations, ARIMA, 14, 63-80, <http://intranet.inria.fr/international/arima/014/014005.html>