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# Stream-potential formulation of atmospheric dynamics in pressure-related coordinates

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Departing from non-hydrostatic covariant equations of motion (1) in isobaric coordinates [1]:

$$\frac{\partial v}{\partial t} = F, \quad v = \{dx/dt, dy/dt, dp/dt\} = \{v^x, v^y, v^p\} = \{u, v, \omega\} \tag{1}$$

equations of motion for 3D velocity divergence $\xi^0$ and curl $\xi$ are derived

$$\frac{\partial \xi^0}{\partial t} = \nabla \cdot F, \quad \frac{\partial \xi}{\partial t} = \nabla \times F, \quad \xi^0 = \nabla \cdot v = \partial_\alpha v^\alpha, \ \ \xi = \nabla \times v \ \ \leftrightarrow \ \ \xi^i = \varepsilon^{i\alpha\beta}\partial_\alpha v_\beta \tag{2}$$

Presenting 3D-velocity **v** in terms of 4D stream-potential $\{\psi^0, \psi_x, \psi_y, \psi_p\}$

$$v = \nabla\psi^0 + \nabla \times \psi \quad \leftrightarrow \quad v^i = G^{i\alpha}\partial_\alpha\psi^0 + \varepsilon^{\alpha\beta\gamma}\partial_\beta\psi_\gamma,$$

where $G^{ij}$ is the metric tensor of pressure ($p$) or hybrid ($\eta$) curved space, subsequent application of **div** and **curl** to **v** yields elliptic equations for $\{\psi^0, \psi_x, \psi_y, \psi_p\}$

$$\mathcal{L}^0\psi^0 = \xi^0, \ \ \mathcal{L}^{i\alpha}\psi_\alpha = \xi^j, \mathcal{L}^0 = \partial_\alpha G^{\alpha\beta}\partial_\beta, \ \ \mathcal{L}^{ij} = \partial_\alpha\left(G^{i\alpha}G^{\beta j} - G^{i\beta}G^{\alpha j}\right)\partial_\beta \tag{3}$$

In native pressure coordinates or for low orography, when G becomes diagonal with main elements

$$G^{11} = \ G^{22} = 1, \quad G^{33} = p^2/H^2, \quad (\ H = RT/g \text{ is the height scale})$$

system (3) can be reduced to three independent equations (4) for three fields – scalar flow potential $\psi^0$, horizontal divergence $\chi = \partial_x\psi_x + \partial_y\psi_y$ of stream function and vertical pressure-velocity $\omega = \partial_x\psi_y - \partial_y\psi_x \equiv dp/dt$:

$$\mathcal{L}^0\psi^0 = \xi^0, \ \ \mathcal{L}^0\chi = \partial_p\left(p^2\xi^p/H^2\right), \mathcal{L}\omega = \frac{p^2}{H^2}\left(\partial_y\xi^x - \partial_x\xi^y\right)\mathcal{L}^0 = \frac{p^2}{H^2}\frac{\partial^2}{\partial p^2} + \nabla^2, \quad \mathcal{L} = \frac{p^2}{H^2}\frac{\partial^2}{\partial p^2} + \nabla^2. \tag{4}$$

As the velocity **v** is specified by the triplet $\omega, \chi, \psi^0$, in full, Eqs. (2) and (4) represent alternate to (1) equations of motion and constitute along with the scale-height (temperature) and surface pressure equations

$$\frac{dH}{dt} = \frac{R}{c_p}\frac{H\omega}{p} + \frac{RQ}{g}, \qquad \frac{\partial p_s}{\partial t} = \nabla \cdot \int_0^{p_s} v dp \tag{5}$$

a complete set for non-hydrostatic modelling of atmospheric dynamics. Equations for $\xi^0, \psi^0, p_s$ represent the 'acoustic loop', which withdraws to the external wave equation for surface pressure fluctuation at slow quasi-hydrostatic motion limit (QHL: $\psi^0 \rightarrow 0$). Equations for $\xi, \chi, \omega, H$ constitute a non-linear, third-order-in-time system for internal inertial-gravity waves, which persists at QHL. Thus, the QHL version of equations can be continued into short scale domain without acoustic component involvement and avoiding the phase distortions of internal waves.

[1] R. Rõõm 1990: General form of non-hydrostatic equations of atmospheric dynamics in isobaric coordinates. Atm. Ocean Physics. v. 26, 17 - 26. [http://meteo.physic.ut.ee/~room/papers/kuni2004/1990.General_Form.pdf ]