



Topological and geometrical analysis of a low-dimensional chaotic model obtained for the dynamics of cereal crops cycles observed from satellite in semi-arid region

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A low-dimensional chaotic model was recently obtained for the dynamics of cereal crops cycles in semi-arid region [1]. This model was obtained from one single time series of vegetation index measured from space. The global modeling approach [2] was used based on powerful algorithms recently developed for this purpose [3]. The resulting model could be validated by comparing its predictability (a data assimilation scheme was used for this purpose) with a statistical prediction approach based on the search of analogous states in the phase space [4].

The cereal crops model exhibits a weakly dissipative chaos ($D_{KY} = 2.68$) and a toroidal-like structure. At present, quite few cases of such chaos are known and these are exclusively theoretical. The first case was introduced by Lorenz in 1984 to model the global circulation dynamics [5], which attractor's structure is remained poorly understood.

Indeed, one very powerful way to characterize low-dimensional chaos is based on the topological analysis of the attractors' flow [6]. Unfortunately, such approach does not apply to weakly dissipative chaos. In this work, a color tracer method is introduced and used to perform a complete topological analysis of both the Lorenz-84 system and the cereal crops model. The usual stretching and squeezing mechanisms are easily detected in the attractors' structure. A stretching taking place in the globally contracting direction of the flow is also found in both attractors. Such stretching is unexpected and was not reported previously. The analysis also confirms the toroidal type of chaos and allows producing both the skeleton and algebraic descriptions of the two attractors. Their comparison shows that the cereal crops attractor is a new attractor.

References

- [1] Mangiarotti S., Drapreau L., Letellier C., 2014. Two chaotic global models for cereal crops cycles observed from satellite in Northern Morocco. revision submitted.
- [2] Letellier C., Aguirre L.A., Freitas U.S., 2009. Frequently asked questions about global modeling. *Chaos*, **19**, 023103.
- [3] Mangiarotti S., Coudret R., Drapreau L., Jarlan L., 2012a. Polynomial Search and Global Modeling – two algorithms for modelling chaos. *Physical Review E*, **86**(4), 046205.
- [4] Mangiarotti S., Mazzega P., Mougin E., Hiernaux P., 2012b. Predictability of vegetation cycles over the semi-arid region of Gourma (Mali) from forecasts of AVHRR-NDVI signals. *Remote Sensing of Environment*, **123**, 246–257.
- [5] Lorenz, 1984. Irregularity: a fundamental property of the atmosphere, *Tellus*, **36A**, 98-110, 1984.
- [6] Gilmore R. & Lefranc M., The topology of chaos, Alice stretch and squeezeland. *Wiley-VCH*, 2002.