



Eulerian Lagrangian Adaptive Fup Collocation Method for solving the conservative solute transport in heterogeneous porous media

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Contaminant transport in natural aquifers is a complex, multiscale process that is frequently studied using different Eulerian, Lagrangian and hybrid numerical methods. Conservative solute transport is typically modeled using the advection–dispersion equation (ADE). Despite the large number of available numerical methods that have been developed to solve it, the accurate numerical solution of the ADE still presents formidable challenges. In particular, current numerical solutions of multidimensional advection-dominated transport in non-uniform velocity fields are affected by one or all of the following problems: numerical dispersion that introduces artificial mixing and dilution, grid orientation effects, unresolved spatial and temporal scales and unphysical numerical oscillations (e.g., Herrera et al, 2009; Bosso et al., 2012).

In this work we will present Eulerian Lagrangian Adaptive Fup Collocation Method (ELAFCM) based on Fup basis functions and collocation approach for spatial approximation and explicit stabilized Runge-Kutta-Chebyshev temporal integration (public domain routine SERK2) which is especially well suited for stiff parabolic problems. Spatial adaptive strategy is based on Fup basis functions which are closely related to the wavelets and splines so that they are also compactly supported basis functions; they exactly describe algebraic polynomials and enable a multiresolution adaptive analysis (MRA). MRA is here performed via Fup Collocation Transform (FCT) so that at each time step concentration solution is decomposed using only a few significant Fup basis functions on adaptive collocation grid with appropriate scales (frequencies) and locations, a desired level of accuracy and a near minimum computational cost. FCT adds more collocations points and higher resolution levels only in sensitive zones with sharp concentration gradients, fronts and/or narrow transition zones.

According to the our recent achievements there is no need for solving the large linear system on adaptive grid because each Fup coefficient is obtained by predefined formulas equalizing Fup expansion around corresponding collocation point and particular collocation operator based on few surrounding solution values. Furthermore, each Fup coefficient can be obtained independently which is perfectly suited for parallel processing. Adaptive grid in each time step is obtained from solution of the last time step or initial conditions and advective Lagrangian step in the current time step according to the velocity field and continuous streamlines. On the other side, we implement explicit stabilized routine SERK2 for dispersive Eulerian part of solution in the current time step on obtained spatial adaptive grid. Overall adaptive concept does not require the solving of large linear systems for the spatial and temporal approximation of conservative transport. Also, this new Eulerian-Lagrangian-Collocation scheme resolves all mentioned numerical problems due to its adaptive nature and ability to control numerical errors in space and time. Proposed method solves advection in Lagrangian way eliminating problems in Eulerian methods, while optimal collocation grid efficiently describes solution and boundary conditions eliminating usage of large number of particles and other problems in Lagrangian methods. Finally, numerical tests show that this approach enables not only accurate velocity field, but also conservative transport even in highly heterogeneous porous media resolving all spatial and temporal scales of concentration field.