



A variational ensemble scheme for noisy image data assimilation

Yin Yang (1), Cordelia Robinson (1), Dominique Heitz (2), and Etienne Mémin (1)

(1) Inria, Campus universitaire de Beaulieu, 35042 Rennes Cedex, France, (2) Irstea, UR TERE, F-35044 Rennes, France

Data assimilation techniques aim at recovering a system state variables trajectory denoted as X , along time from partially observed noisy measurements of the system denoted as \mathcal{Y} . These procedures, which couple dynamics and noisy measurements of the system, fulfill indeed a twofold objective. On one hand, they provide a denoising – or reconstruction – procedure of the data through a given model framework and on the other hand, they provide estimation procedures for unknown parameters of the dynamics. A standard variational data assimilation problem can be formulated as the minimization of the following objective function with respect to the initial discrepancy, η , from the background initial guess:

$$J(\eta(x)) = \frac{1}{2} \|X_0^b(x) - X(t_0, x)\|_B^2 + \frac{1}{2} \int_{t_0}^{t_f} \|\mathbb{H}(X(t, x)) - \mathcal{Y}(t, x)\|_R^2 dt. \quad (1)$$

where the observation operator \mathbb{H} links the state variable and the measurements. The cost function can be interpreted as the log likelihood function associated to the *a posteriori* distribution of the state given the past history of measurements and the background. In this work, we aim at studying ensemble based optimal control strategies for data assimilation. Such formulation nicely combines the ingredients of ensemble Kalman filters and variational data assimilation (4DVar). It is also formulated as the minimization of the objective function (1), but similarly to ensemble filter, it introduces in its objective function an empirical ensemble-based background-error covariance defined as:

$$B \approx \langle (X^b - \langle X^b \rangle) (X^b - \langle X^b \rangle)^T \rangle. \quad (2)$$

Thus, it works in an off-line smoothing mode rather than on the fly like sequential filters.

Such resulting ensemble variational data assimilation technique corresponds to a relatively new family of methods [1,2,3]. It presents two main advantages: first, it does not require anymore to construct the adjoint of the dynamics tangent linear operator, which is a considerable advantage with respect to the method's implementation, and second, it enables the handling of a flow-dependent background error covariance matrix that can be consistently adjusted to the background error. These nice advantages come however at the cost of a reduced rank modeling of the solution space. The B matrix is at most of rank $N - 1$ (N is the size of the ensemble) which is considerably lower than the dimension of state space. This rank deficiency may introduce spurious correlation errors, which particularly impact the quality of results associated with a high resolution computing grid. The common strategy to suppress these distant correlations for ensemble Kalman techniques is through localization procedures.

In this paper we present key theoretical properties associated to different choices of methods involved in this setup and compare with an incremental 4DVar method experimentally the performances of several variations of an ensemble technique of interest. The comparisons have been led on the basis of a Shallow Water model and have been carried out both with synthetic data and real observations. We particularly addressed the potential pitfalls and advantages of the different methods. The results indicate an advantage in favor of the ensemble technique both in quality and computational cost when dealing with incomplete observations. We highlight as the premise of using ensemble variational assimilation, that the initial perturbation used to build the initial ensemble has to fit the physics of the observed phenomenon. We also apply the method to a stochastic shallow-water model which incorporate an uncertainty expression if the subgrid stress tensor related to the ensemble spread.

References

- [1] A. C. Lorenc, The potential of the ensemble kalman filter for nwp — a comparison with 4d-var, *Quart. J. Roy. Meteor. Soc.*, Vol. 129, pp. 3183–3203, 2003.
- [2] C. Liu, Q. Xiao, and B. Wang, An Ensemble-Based Four-Dimensional Variational Data Assimilation Scheme. Part I: Technical Formulation and Preliminary Test, *Mon. Wea. Rev.*, Vol. 136(9), pp. 3363–3373, 2008.

- [3] M. Buehner, Ensemble-derived stationary and flow-dependent background-error covariances: Evaluation in a quasi- operational NWP setting, *Quart. J. Roy. Meteor. Soc.*, Vol. 131(607), pp. 1013–1043, April 2005.