



## Mixtures and Kernel models of Inference for Coherent Structures

Piyush Tagade and Sai Ravela

Massachusetts Institute of Technology, Earth, Atmospheric and Planetary Sciences, Cambridge, United States  
(tagade@mit.edu)

Coherent structures emerge ubiquitously in flows, yet little has been done to account for or exploit the attendant pattern information when solving inference problems associated with geophysical fluids.

Using field alignment and field coalescence we show how accounting for pattern phase can benefit state estimation and uncertainty quantification. This unambiguously motivates a joint position amplitude representation for inference in pattern space.

We then present a new framework for coherent structures using a new formulation for mixtures and kernels, respectively the Mixture-Ensemble Kalman Filter (MEnKF) and Mutual Information Filter ( $\mu$ -IF), both of which are usable in variational and probabilistic settings of inference.

The MEnKF advances current state of the art by replacing the implicit Gaussian approximation of forecast ensemble in EnKF using a Gaussian mixture model, that can approximate any probability density function to the arbitrary accuracy through the proper choice of number of mixtures.

The MEnKF finds natural application in coherent structures, where the emergent patterns are approximated using a sum of mixtures, and whose evolution is modeled through appropriate updates in the mixture weights.

We discuss additional generalizations in the form of mutual information, where patterns explored through the ensemble simulations are approximated using a localized Gaussian kernel density estimator. We finally present an inference framework based on the maximization of the mutual information that is formulated through a kernel density estimate of the Renyi entropy.

We investigate efficacy of the proposed formulations for data assimilation in the context of solitons, obtained as a solution to the Korteweg-de Vries equation.

Using a soliton mixture to illustrate, we demonstrate the importance of accounting for phase in geophysical inference.

We then generalize our approach by utilizing a phase-amplitude parametrization that demonstrate good performance on chaotic solitons, providing a well-tracked quadratic estimation procedure.