Geophysical Research Abstracts
Vol. 16, EGU2014-1359, 2014
EGU General Assembly 2014

To the Calculation of Waves Refraction at the Shore Slope Having Arbitrary Steepness

Shalva Gagoshidze
Georgian Technical University, Civil Engineering Department, Tbilisi, Georgia (sh.gagoshidze@gmail.com)

A three-dimensional problem of linear waves refraction near the bank slope with an arbitrary angle tilt to the horizon ($0 < \theta_0 \leq \pi$) is solved in a cylindrical coordinate system. On choosing a basis function satisfying exactly all boundary conditions (as distinct from the Berkhoff basis function [1]) and averaging the wave equations by the direct Kantorovich method [2] with respect to the polar angle θ, we derive a one-dimensional evolutionary equation of Schrodinger type which is asymptotically solved by the WKB method [3] for an arbitrary angle between the wave crests on the deep water and the shore line $0^\circ \leq \alpha_0 < 90^\circ$. The final results are easily observable as different from extremely complicated solutions of Peters, Rose, Stoker and others [4]. In particular, for the wave amplitude transformation coefficient we obtain

$$K_a = \frac{a}{a_0} = \frac{(k/k_0)\sqrt{\cos\alpha_0}}{(1+2kr\theta_0/\sinh 2kr\theta_0)^{1/2}}[(k/k_0)^2 + (R-N)\theta_0^2 - \sin^2\alpha_0]^{-1/4}; \quad (1)$$

$_{Here}k_0$ is a wave number given at infinity (given in fact at a dimensionless distance from the shore approximately equal to $(6 \div 7)k_0 r\theta_0$); the variable parameter k is defined from the transcendent equation

$$k\tanh kr\theta_0 = k_0 \quad ; \quad (2)$$

the symbols R and Ndenote the expressions

$$R = \frac{1}{4(k_0 r\theta_0)^2} \quad ; \quad (3)$$

$$N = \frac{(k/k_0)^2}{(1+\frac{2kr\theta_0}{\sinh 2kr\theta_0})^3}\left[1 - \frac{\coth 2kr\theta_0}{kr\theta_0} + \frac{1}{2(kr\theta_0)^2} - \frac{2kr\theta_0}{3\sinh 2kr\theta_0} + \frac{(\coth 2kr\theta_0 - 1/2kr\theta_0)^2}{1+2kr\theta_0/\sinh 2kr\theta_0}\right], \quad (4)$$

According to the WKB approximation requirements for Schrodinger equations [3], the term $(R-N)\theta_0^2$ in (1) should be neglected for relatively small $k_0 r\theta_0$ (for $k_0 r\theta_0 \leq 2$ by our estimation). For quite small θ_0 this term should be completely neglected. In that case relation (1) exactly coincides with an analogous relation derived by the energy-optical method.

An almost complete coincidence of the obtained results with the results of separate numerical realizations of the above-mentioned exact solutions given in [4] is observed also in the case of waves refraction near steeply and negative tilted shore. Moreover, as follows from the proposed model, when $75^\circ \leq \alpha_0 < 90^\circ$ and $\frac{\pi}{4} \leq \theta_0 \leq \pi$, the occurrence of the so-called "potential well" and wave phase shift is excellently confirmed by the well-known Pierson's photo of refraction of real waves ([4], Fig 5.6.2).

References

1. Berkhoff J.C.W. Mathematical models for simple harmonic linear waves. Wave diffraction and refraction // Delft University of Technology: Publ. No. 163, 1976, 108 p.

2. Kantorovich L.V. and Krylov V.I. Methods for the approximate solution of partial differential equations (review) // Moscow, J. Uspekhi Mat. Nauk, No. 5, 1938, pp.263–265.

3. Nikiforov A.F., Uvarov V.B. Special Functions of Mathematical Physics // Moscow, Nauka,1984, 344 p.

4. Stoker J.J. Water waves. The mathematical theory with applications // New York: Interscience Publishers, 1957. 567 p.

The work was supported by Grant 10-15 of the Georgian National Science Foundation.