



## A study on parameters for implicit penalizing-load algorithms for stabilization of free surfaces in geodynamic models

Miguel Andres-Martinez (1), Jason Phipps Morgan (1), Marta Perez-Gussinye (1), and Lars H. Ruepke (2)

(1) Royal Holloway University of London, Earth Sciences, Egham, United Kingdom, (2) GEOMAR | Helmholtz Centre for Ocean Research, Kiel, Germany

Earth surface in geodynamic codes has been typically modeled as a free-stress surface, which means that the total stresses applied to the surface are considered zero value. Free surface allows topography to be generated in response to the inner forces of geodynamic models. Surficial processes, such as erosion and sedimentation, reshape topography and, therefore, change the loads to be considered in the geodynamic processes. Therefore, a free surface is also needed to couple modeling of surficial processes together with mantle dynamics. Real free surface models also allow to study relaxation of topography, such as isostatic post-glacial rebound, in a geodynamic setting. However, free surfaces have typically confronted stability problems when the time step chosen to run the model is bigger than the viscous relaxation time. Time steps small enough to avoid free-surface instabilities usually result in computationally expensive models. We have developed a free-surface stabilization algorithm (FSSA) to avoid instabilities for bigger time steps ( $>10$  Kyr), which penalizes the system with a load calculated implicitly from a portion of the difference between the topography for an initial time step, and the future topography in the next time step. The penalization load is formulated based on the velocities at the nodes for the beginning of the time step, and applied to the nodes at the beginning of the same time step. Additionally, we have coded the FSSA described in Kaus, Mühlhaus and May (2010), for comparison. Their algorithm also penalizes the system with a load which is calculated deriving the surface traction terms from the time discretization of the momentum equation. The penalizing terms in both our and Kaus' FSSAs are controlled by a factor which value ranges between 0 and 1. Several simplistic viscous tests have been run in order to find the optimal (more accurate and more stable) control factor for both algorithms. These tests showed that both methods produce very similar and accurate results (error  $\sim <0.01$ ), and that 2/3 is the optimal control factor for together stability and accuracy, for both FSSAs. The penalizing terms account for a vertical component and an horizontal component, which are symmetric and asymmetric in Stiffness-shape form, respectively. Consequently, the horizontal component of the penalization makes the Stiffness matrix to be asymmetric, so Cholesky factorization cannot be applied, which results into a slower solution of the system. We have developed a scheme to apply Cholesky factorization to the symmetric terms, and use Uzawa-like iterations to include the asymmetric terms in the system. This should intuitively give more stable results than FSSAs that omit the horizontal penalization terms, especially for topographies with steep slopes, where the horizontal component of the velocities is an important term for the surface displacement.