



## External and internal waves in stream-potential pressure-coordinate dynamics

Marko Zirk and Rein Rõõm

University of Tartu, Laboratory of Atmospheric Physics, Tartu, Estonia (marko.zirk@ut.ee)

In stream-potential dynamics pressure coordinate velocity

$\vec{v} = \{dx/dt, dy/dt, dp/dt\} = \{v^x, v^y, v^p\} = \{u, v, \omega\}$  is presented in terms of 4D stream-potential  $\{\psi^0, \psi_x, \psi_y, \psi_p\}$

$$\vec{v} = \nabla\psi^0 + \nabla \times \vec{\psi} \leftrightarrow v^i = G^{i\alpha} \partial_\alpha \psi^0 + \varepsilon^{\alpha\beta\gamma} \partial_\beta \psi_\gamma,$$

with diagonal metric tensor with main elements  $G^{11} = G^{22} = 1, G^{33} = p^2/H^2$  ( $H = RT/g$  is the height scale). Vector potential  $\vec{\psi}$  is further expressed via horizontal curl and divergence of the stream function

$$\omega = \partial_x \psi_y - \partial_y \psi_x \equiv dp/dt, \quad \chi = \partial_x \psi_x + \partial_y \psi_y.$$

The wave-vector components in linearized stream-potential dynamics are the scalar flow potential  $\psi^0$ , surface pressure fluctuation  $p'_s$ , horizontal divergence  $\chi$  and curl  $\omega$  of the fluctuative part  $\vec{\psi}'$  of complete vector potential  $\vec{\psi} = \vec{\bar{\psi}} + \vec{\psi}'$ .

Equations for  $\psi^0$  and  $p'_s$  form the external wave subsystem

$$\frac{\partial p'_s}{\partial t} + \nabla^2 \int_0^{\bar{p}_s} \psi^0 dp = 0, \quad \frac{d\xi^0}{dt} + gH \nabla^2 (p'_s/\bar{p}_s) = A^0(\vec{\bar{\psi}}, \omega, \chi), \quad \mathcal{L}^0 \psi^0 = \xi^0, \quad \mathcal{L}^0 = \frac{p^2}{H^2} \frac{\partial^2}{\partial p^2} + \nabla^2, \quad (1)$$

while the equations for  $\chi, \omega$  and temperature fluctuation  $T'$  form the internal wave subsystem

$$\frac{d\vec{\xi}}{dt} = \vec{A}(\vec{\bar{\psi}}, \omega, \chi), \quad \frac{dT'}{dt} = \frac{T_i \omega}{p} + Q, \quad \mathcal{L}^0 \chi = \partial_p (p^2 \xi^p / H^2), \quad \left( \frac{p^2}{H^2} \frac{\partial^2}{\partial p^2} + \nabla^2 \right) \omega = \frac{p^2}{H^2} (\partial_y \xi^x - \partial_x \xi^y) \quad (2)$$

with  $T_i = (R/c_p)\bar{T} - p\partial_p \bar{T}$ . In these equations

$$\xi^0 = \nabla \cdot \vec{v} = \partial_\alpha v^\alpha, \quad \vec{\xi} = \nabla \times \vec{v} \leftrightarrow \xi^i = \varepsilon^{i\alpha\beta} \partial_\alpha v_\beta$$

are the 3D divergence and curl of velocity.

In the presentation equation systems (1) and (2) are solved both analytically and numerically. Interaction of external waves with stationary internal orographic waves is investigated.