



## Triadic Non-Gaussian low-frequency Teleconnections in the Atmosphere and Ocean

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Teleconnections (TCs) normally rely upon long distance and simultaneous/lagged non-zero correlations of a geophysical field. However, for non-Gaussian multivariate probability distributions like that of the space spanned by low-frequency atmospheric-oceanic components, the linear Pearson correlation can be zero whereas some nonlinear correlation is nonzero. Therefore, in the non-Gaussian world, TCs can only be correctly assessed by the multiinformation (MI) - generalization of mutual information for any number of variables.

Non-Gaussianity of spatially-distributed geophysical complex networks, still allows for a more 'exotic' behavior (triads), where three variables  $X, Y, Z$  (e.g. field values taken at three mutually distant points) are uncorrelated or even pair-wised statistically independent, (i.e. vanishing mutual information  $I(X, Y) = I(X, Z) = I(Y, Z) = 0$ ), while the triadic MI  $I(X, Y, Z)$  is greater than zero. These 'threesome' TCs (perfect and almost perfect triads) are shown to exist in the atmospheric-oceanic monthly-decadal timescale variability. In order to show that, two datasets are used: a) monthly-running averages of the stream-function fields issued from a million-day run of a quasi-geostrophic 3-level, T21 model (QG-model); b) annual-running SST averages for the 1880-2012 period, taken from GISS. The intensity of triadic TCs are measured by the interaction multiinformation (IMI)  $II(X, Y, Z) = I(X, Y, Z) - [I(X, Y) + I(X, Z) + I(Y, Z)]$  which is positive (negative) in case of synergy (redundancy) among variables. A relevant remark is the fact that the coarse-grained IMI version is maximal when the categorical variable outcomes satisfy a Latin-Square relationship (e.g. the Boolean exclusive disjunction of 2 symbols, i.e.  $Z = X \text{ and/or } Y$ , the Sudoku game of 9 symbols).

We devise an optimization gradient-descent-based algorithm for finding triads in the space of orthogonally rotated normalized principal components (RN-PCs) of the analyzed field. RN-PCs  $(X, Y, Z)$  are uncorrelated by construction, and thus the pair-wised MIs are close to zero as wished. Moreover, rotations are restricted to the few (Nrot) leading variance-explaining PCs assuring that the sub-space spanned by triads do not project onto noisy components. In order to accomplish that, we find local maxima (and then the absolute maximum) of a proxy-functional  $F(X, Y, Z)$  of  $II(X, Y, Z)$ , for instance the monomial expectation  $E(XYZ)$ , which is proportional to the correlation between a product of two arbitrary chosen variables with the third one. The control vector is filled up by the  $Nrot(Nrot-1)/2$  Euler angles spanning all the possible orthogonal rotations on a Nrot-dimensional metric space. That proxy-functional relies upon the set of previously estimated third order moments among the unrotated N-PCs. Positive  $E(XYZ)$  means that, either the three time-series of  $X, Y, Z$  are positively-phased or, two out of the three variables are negatively-phased with the third one positively-phased.

As regards the dominant triads, maximizing  $F(X, Y, Z)$  for a given Nrot dimension, we compute temporal correlation maps between either  $X, Y$  or  $Z$  and the original field. The corresponding spatial patterns project mainly on Rossby waves (maybe in triadic resonance) for the case of the QG-model. For the case of interannual SST variability, we find patterns, somehow projected onto the El-Niño, AMO, PDO and other patterns. The main triadic interacting SST patterns seem to be: 1) The zone of Atlantic and Pacific cold currents; 2) The California current zone; 3) The Equatorial Counter-current zone.

The teleconnecting triads may be useful for characterizing the nonlinear non-Gaussian variability as well as for studies of long-range predictability.

Work supported by the Portuguese Science Foundation (FCT) under the project PTDC/GEO-MET/3476/2012.