



Inertial Wave Excitation and Wave Attractors in an Annular Tank: DNS

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Rotation is the most relevant aspect of geophysical fluid dynamics, manifesting itself by the Coriolis force. Small perturbations to the state of rigid body rotation can excite inertial waves (waves restored by Coriolis force) with frequencies in the range $0 < \sigma < 2\Omega_0$. We can restrict our attention to an incompressible fluid so that inertial waves remain the only waves in the mathematical model, which can transport kinetic energy and angular momentum. In geophysics, inertial waves have received a lot of attention over the last decades. A spherical shell, for instance, is already non-simple in a sense that its inertial mode's spatial structures are complex, forming so-called wave attractors [1]. But also other containers have been investigated, e.g., cylinders and boxes from the viewpoints of normal mode excitation [2,3], mean flow generation and boundary layer flow [4]. A simple wave attractor was found in a prism, which can be seen as idealized ocean basin [5]. However, local mechanisms of wave excitation are still not very well understood.

In order to contribute to the ongoing discussion, we consider an annular geometry. Its rectangular symmetry was broken by replacing the inner cylinder with a frustum of apex half-angle $\alpha = 5.7^\circ$. The annular gap is filled with a fluid of kinematic viscosity ν . The whole vessel rotates with a mean angular velocity Ω_0 around its axis of symmetry. Ekman numbers investigated are $1 \gg E = \nu(\Omega_0 H^2)^{-1} \geq 10^{-5}$. Similarly to [1–5] we perturb the system by longitudinal libration, $\Omega(t) = \Omega_0(1 + \varepsilon \sin \omega t)$, where $\omega > 0$ denotes the frequency and $0 < \varepsilon < 1$ the amplitude of libration.

Three-dimensional direct numerical simulations (3-D DNS) of the set-up were conducted in order to resolve different excitation mechanisms. We used an incompressible Navier–Stokes solver with the equations formulated for volume fluxes in generalized curvilinear coordinates. Under some constraints the scheme conserves three quantities of Hamiltonian mechanics: mass, momentum and kinetic energy. To separate between possible excitation mechanisms we investigated configurations that cannot be accessed in the laboratory, e.g., axially periodic geometries and cases with libration of different walls.

For $\varepsilon \leq 0.3$ we found qualitative agreement of wave attractor patterns obtained by numerical simulations, ray tracing and measurements in the laboratory for all libration frequencies investigated. We adapted boundary layer theory for the librating walls to estimate inertial wave excitation, in particular, the relation to libration frequency and amplitude, as well as the effect of the inclination angle α of the frustum. By comparison with numerical simulations we found that wave energy in the bulk obeys a similar dependency on frequency as the energy in the boundary layer over the librating wall.

References

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