



Mathematical apparatus for boundary value problems in gravity field studies and the geometry of the solution domain

Petr Holota (1) and Otakar Nesvadba (2)

(1) Research Institute of Geodesy, Topography and Cartography, 250 66 Zdíby 98, Praha-vychod, Czech Republic, e-mail: holota@pecny.asu.cas.cz, Tel.: +420 323649235, Fax: +420 284890056, (2) Land Survey Office, Pod Sídlištěm 9, 182 11 Praha 8, Czech Republic. e-mail: nesvadba@sky.cz

In geodesy mathematical techniques for gravity field studies that rest on the concept of the so-called classical solution of boundary value problems, have a rather traditional position. Nevertheless, the range of the tools for treating problems in this field is much wider. For instance the concept of the weak solution met with a considerable attention. From this point of view the approach is associated with constructing the respective integral kernels or Green's function in case we consider the classical solution concept or with the choice and constructing basis functions in case we are looking for the weak solution of the problem. Within the tools considered we discuss also the use of reproducing kernels. In both the cases (classical or weak) the construction of the apparatus above represents an important technical step. It is not elementary, but for a number of fundamental boundary value problems the solution is known, in particular in the case of a spherical solution domain. The sphere, however, is rather far from the real shape of the Earth, which is interpreted here in terms of a functional analytic norm. The distance has a negative effect on any attempt to reach the solution of the boundary value problems considered (and to bridge the departure of the Earth's surface from the sphere) by an iteration procedure based on a successive application of a solution technique developed for the spherical boundary. From this point of view the construction of the integral kernels and basis functions for an oblate ellipsoid of revolution means a step closer towards reality. In this contribution we on the one hand give an overview of the results already achieved and subsequently develop the topic. The summation of series of ellipsoidal harmonics is one of the key problems in this connection. Hypergeometric functions and series are applied too. We also show where the use of Legendre elliptic integrals adds to the solution of the problem. It is interesting that they do not appear in the ellipsoidal case only, but also in constructing the apparatus related to the spherical boundary. The computation of the integrals is treated too. Finally, we discuss the compensation of a small violation of harmonicity caused by approximations applied in the technical implementation of the apparatus and pay also attention to singularity problems when integral kernels or basis functions are applied. The use of the tools above is discussed and demonstrated by numerical examples.