



P-adic model of transport in porous disordered media

Adrei Yu. Khrennikov (1) and Klaudia Oleschko (2)

(1) International Center for Mathematical Modelling in Physics and Cognitive Sciences Linnaeus University, SE-351 95, Sweden, (2) Centro de Geociencias Universidad Nacional Autonoma de Mexico. Campus UNAM Juriquilla, Queretaro, C.P. 76230, Mexico.

The soil porosity and permeability are the most important quantitative indicators of soil dynamics under the land-use change. The main problem in the modeling of this dynamic is still poor correlation between the real measuring data and the mathematical and computer simulation models. In order to overcome this deep divorce we have designed a new technique, able to compare the data arising from the multiscale image analyses and time series of the basic physical properties dynamics in porous media studied in time and space. We present a model of the diffusion reaction type describing transport in disordered porous media, e.g., water or oil flow in a complex network of pores. Our model is based on p-adic representation of such networks. This is a kind of fractal representation. We explore advantages of p-adic representation, namely, the possibility to endow p-adic trees with an algebraic structure and ultrametric topology and, hence, to apply analysis which have (at least some) similarities with ordinary real analysis on the straight line. We present the system of two diffusion reaction equations describing propagation of particles in networks of pores in disordered media. As an application, one can consider water transport through the soil pore networks, or oil flow through capillaries nets. Under some restrictions on potentials and rate coefficients we found the stationary regime corresponding to water content or concentration of oil in a cluster of capillaries. Usage of p-adic analysis (in particular, p-adic wavelets) gives a possibility to find the stationary solution in the analytic form which makes possible to present a clear pedological or geological picture of the process. The mathematical model elaborated in this paper (Khrennikov, 2013) can be applied to variety of problems from water concentration in aquifers to the problem of formation of oil reservoirs in disordered media with porous structures. Another possible application may have real practical output. In fact, our system of diffusion-reaction equations can be used to model the process of extraction of water or oil from an extended network of capillaries (Khrennikov et al., 2013). The accomplished analyses show that the time series of water content/pressure dynamics in saturated/unsaturated conditions reflect the fractal structure of pores separated by families based on the seven geometric descriptors which we used for the soils multiscale images (Oleschko et al., 2012). The similar models were applied to the porous media behind the oil flow from wells. These results motivate usage of the fractal and, in particular, p-adic methods of modeling.