



## Failure-plane angle in Bentheim sandstone subjected to true triaxial stresses: experimental results and theoretical prediction

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We conducted true triaxial tests in the high-porosity ( $n = 24\%$ ), quartz-rich (95%), Bentheim sandstone. An important objective was to investigate the dependence of failure-plane angle  $\theta$  (angle between the normal to the plane and  $\sigma_1$  direction) on the prevailing stress conditions. We employed two distinct loading paths, and seven  $\sigma_3$  magnitudes (between 0 and 150 MPa). In tests using the common loading path,  $\sigma_2$  and  $\sigma_3$  were fixed, while  $\sigma_1$  was raised monotonically to failure. In tests using the novel loading path (which facilitate comparison with theoretical predictions),  $\sigma_3$  was fixed, and the Lode angle,  $\Theta (= \tan^{-1} [(\sigma_1 - 2\sigma_2 + \sigma_3) / 3^{0.5}(\sigma_1 - \sigma_3)])$  was kept constant by raising  $\sigma_1$  and  $\sigma_2$  simultaneously at a set ratio  $b [= (\sigma_2 - \sigma_3) / (\sigma_1 - \sigma_3)]$  until failure occurred. Six stress ratios  $b (= 0, 1/6, 1/3, 1/2, 3/4, 1)$ , i.e. six  $\Theta (= \tan^{-1} [(1-2b) / 3^{0.5}])$  values from  $+\pi/6$  (axisymmetric compression) to  $-\pi/6$  (axisymmetric extension) were used.

In axisymmetric common loading path tests, failure-plane angle  $\theta$  generally declined as the applied  $\sigma_3 = \sigma_2$  increased from about  $80^\circ$  at  $\sigma_3 = \sigma_2 = 0$  MPa to  $0^\circ$  at  $\sigma_3 = \sigma_2 = 150$  MPa (forming compaction bands). In tests where  $\sigma_3 \neq \sigma_2$ , the resulting failure-plane strike was consistently parallel to  $\sigma_2$  direction. For low  $\sigma_3$ ,  $\theta$  typically rose by up to  $12^\circ$  as  $\sigma_2$  rose from  $\sigma_2 = \sigma_3$  to  $\sigma_2 = \sigma_1$ . However, the rise in  $\theta$  with  $\sigma_2$  tended to diminish at higher  $\sigma_3$ . A limiting case occurred at  $\sigma_3 = 150$  MPa, where failure plane remained at  $0^\circ$ , regardless of the rise in  $\sigma_2$ . In the novel loading path tests, failure-plane angle  $\theta$  declined monotonically for any given Lode angle  $\Theta$ , from roughly  $80^\circ$  to  $0^\circ$ , as the mean stress at failure ( $\sigma_{oct,f}$ ) rose from about 20 MPa to around 220 MPa; for a constant  $\sigma_{oct,f}$ ,  $\theta$  typically increased from  $10^\circ$  (at  $\sigma_{oct,f} = 20$  MPa) to  $30^\circ$  (at  $\sigma_{oct,f} = 220$  MPa) as  $\Theta$  dropped from  $+\pi/6$  ( $\sigma_2 = \sigma_3$ ) to  $-\pi/6$  ( $\sigma_2 = \sigma_1$ ).

We compared the measured  $\theta$  with that predicted using equation 28 in Rudnicki (2013), an extension of the Rudnicki and Rice (1975) prediction to include the third stress invariant  $\Theta$ . (Space does not permit detailing the equation in this abstract.) The theory treats octahedral shear stress at failure ( $\tau_{oct,f}$ ) and the resulting  $\theta$  as dependent on  $\sigma_{oct,f}$  and  $\Theta$ . We used two series of the novel loading path tests: axisymmetric compression ( $\Theta = +\pi/6$ ) and pure shear ( $\Theta = 0$ ) to constrain that dependence. The failure conditions in the novel loading path tests were then simulated to compare the predicted failure-plane angles with the experimental results. The predictions were in general agreement with the experimental data, except when  $\Theta = -\pi/6$  ( $\sigma_2 = \sigma_1$ ). In the common loading path tests, failure prediction replicated the general rise of the experimentally observed  $\theta$  with  $\sigma_2$  for a given  $\sigma_3$ , as well as the diminished rise at high  $\sigma_3$  magnitudes.

The reasonable agreement between the predicted and the observed failure-plane angle demonstrated the applicability and the limitations of Rudnicki's (2013) theory.