



Failure-plane angle in Bentheim sandstone subjected to true triaxial stresses: experimental results and theoretical prediction

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We conducted true triaxial tests in the high-porosity ($n = 24\%$), quartz-rich (95%), Bentheim sandstone. An important objective was to investigate the dependence of failure-plane angle θ (angle between the normal to the plane and σ_1 direction) on the prevailing stress conditions. We employed two distinct loading paths, and seven σ_3 magnitudes (between 0 and 150 MPa). In tests using the common loading path, σ_2 and σ_3 were fixed, while σ_1 was raised monotonically to failure. In tests using the novel loading path (which facilitate comparison with theoretical predictions), σ_3 was fixed, and the Lode angle, $\Theta (= \tan^{-1} [(\sigma_1 - 2\sigma_2 + \sigma_3) / 3^{0.5}(\sigma_1 - \sigma_3)])$ was kept constant by raising σ_1 and σ_2 simultaneously at a set ratio $b [= (\sigma_2 - \sigma_3) / (\sigma_1 - \sigma_3)]$ until failure occurred. Six stress ratios $b (= 0, 1/6, 1/3, 1/2, 3/4, 1)$, i.e. six $\Theta (= \tan^{-1} [(1-2b) / 3^{0.5}])$ values from $+\pi/6$ (axisymmetric compression) to $-\pi/6$ (axisymmetric extension) were used.

In axisymmetric common loading path tests, failure-plane angle θ generally declined as the applied $\sigma_3 = \sigma_2$ increased from about 80° at $\sigma_3 = \sigma_2 = 0$ MPa to 0° at $\sigma_3 = \sigma_2 = 150$ MPa (forming compaction bands). In tests where $\sigma_3 \neq \sigma_2$, the resulting failure-plane strike was consistently parallel to σ_2 direction. For low σ_3 , θ typically rose by up to 12° as σ_2 rose from $\sigma_2 = \sigma_3$ to $\sigma_2 = \sigma_1$. However, the rise in θ with σ_2 tended to diminish at higher σ_3 . A limiting case occurred at $\sigma_3 = 150$ MPa, where failure plane remained at 0° , regardless of the rise in σ_2 . In the novel loading path tests, failure-plane angle θ declined monotonically for any given Lode angle Θ , from roughly 80° to 0° , as the mean stress at failure ($\sigma_{oct,f}$) rose from about 20 MPa to around 220 MPa; for a constant $\sigma_{oct,f}$, θ typically increased from 10° (at $\sigma_{oct,f} = 20$ MPa) to 30° (at $\sigma_{oct,f} = 220$ MPa) as Θ dropped from $+\pi/6$ ($\sigma_2 = \sigma_3$) to $-\pi/6$ ($\sigma_2 = \sigma_1$).

We compared the measured θ with that predicted using equation 28 in Rudnicki (2013), an extension of the Rudnicki and Rice (1975) prediction to include the third stress invariant Θ . (Space does not permit detailing the equation in this abstract.) The theory treats octahedral shear stress at failure ($\tau_{oct,f}$) and the resulting θ as dependent on $\sigma_{oct,f}$ and Θ . We used two series of the novel loading path tests: axisymmetric compression ($\Theta = +\pi/6$) and pure shear ($\Theta = 0$) to constrain that dependence. The failure conditions in the novel loading path tests were then simulated to compare the predicted failure-plane angles with the experimental results. The predictions were in general agreement with the experimental data, except when $\Theta = -\pi/6$ ($\sigma_2 = \sigma_1$). In the common loading path tests, failure prediction replicated the general rise of the experimentally observed θ with σ_2 for a given σ_3 , as well as the diminished rise at high σ_3 magnitudes.

The reasonable agreement between the predicted and the observed failure-plane angle demonstrated the applicability and the limitations of Rudnicki's (2013) theory.