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Mathematical analysis of the Saint-Venant-Hirano model and numerical solution by path-conservative methods.

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River beds are usually characterised by the presence of mixed sediment, i.e. sediments of different size.
The mathematical description of mixed-sediment transport requires a proper sediment continuity model, relating bedload transport to the size of sediments available at the bed surface and keeping track of the development of stratigraphy. Hirano[1971,1972] was the first to develop such a model. He discretised the bed material using a finite number of classes characterized by a unique grain size value and put forward the concept of active layer, i.e. a fully-mixed layer located just under the bed surface. The active layer represents the volume of sediments that interact with the flow giving rise to bedload fluxes and regulates the exchange with the substrate located underneath.

We study the system of governing equations given by the active layer equations coupled with the Saint-Venant model for free-surface shallow flow, in one space dimension. After rewriting the problem in matrix-vector form (Stecca et al, submitted), we show that, with respect to the Saint-Venant-Exner model for homogeneous sediment, the introduction of N active layer equation adds N positive characteristic directions, denoting distinct downstream-travelling waves.
We investigate the role of different waves in advecting morphodynamic changes through the domain. To this aim, we implement an analytical linearised solver to analyse the propagation of small-amplitude
perturbations of the bed elevation and grainsize distribution of the active layer. We find that initial gradients in the grainsize distribution of the active layer are able to trigger significant bed variations, that propagate in the downstream direction at faster pace than the bed wave arising from the uniform-sediment Saint-Venant-Exner model. We also verify that multiple sorting waves induce multiple associated bed perturbations, travelling at different speeds.

Finally, we numerically solve the system of governing equations using a synchronous approach, by which all the variables are updated simultaneously. The non-conservative problem which stems from the developed matrix-vector formulation is solved using path-conservative methods. We perform numerical applications by comparison with the above linearised solutions and with the data from laboratory experiments. Results show that our solution approach is robust, general and accurate.

References

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