



Deterministic models for Laplacian growth of thin fingers in a Hele-Shaw geometry

Michał Pecelerowicz and Piotr Szymczak

University of Warsaw, Institute of Theoretical Physics, Faculty of Physics, Warsaw, Poland (pecel@fuw.edu.pl)

Laplacian growth is one of the fundamental mechanisms of pattern formation, driving such natural processes like electrodeposition, dielectric breakdown or viscous fingering. The characteristic features of these processes include a strong competition between spontaneously formed dendrite-like structures, and tip-splitting effects when dendrites bifurcate into secondary branches.

Following Carleson and Makarov [1] we consider a simple, deterministic model of Laplacian growth, in which the growth takes place only at the tips of the long-and-thin fingers. The dynamics of the system can then be described by a deterministic Loewner equation. However, in contrast to [1], we allow the fingers to split, which is crucial to obtain the patterns corresponding to those observed in nature. We discuss different splitting criteria and study the dynamics of the model in variety of geometries including half-plane, channel, radial and the Hele-Shaw geometry.

The latter is particularly relevant experimentally. In the Hele-Shaw cell, the pressure difference is imposed at two opposite ends of the system, whereas the two other boundaries are assumed to be reflecting. We derive the Loewner equation for this geometry (expressed in terms of elliptic functions) and study the tip-splitting dynamics in such a setting. In particular, we analyze in detail the tip-splitting cascade, triggered as the fingers approach the edge of the system.

[1] L. Carleson and N. Makarov, *J. Anal. Math.* 87, 103 (2002).