



## Refraction of the principal stress trajectories at a surface of discontinuity and related problems

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The principal stress trajectories (PST) obtainable from the inversion of data on various types of coeval natural stress indicators provide important information on the real stress fields of the Earth's crust. In particular, once the PST field has been obtained, the stress field can be calculated from equilibrium conditions without regard to the crust rheological properties. In the case under consideration the equilibrium conditions form a closed hyperbolic system of differential equations on unknown magnitudes of the principal stresses. The PST concept is significantly complicated in presence of faults, natural interfaces, and other surfaces of discontinuity where according to laboratory experiments and drilling results the stress trajectories are usually refracted. Unfortunately, the phenomenon of the PST refraction is poorly understood theoretically and often ignored in mathematical modelling which leads to unjustified conclusions.

The full investigation of the phenomenon has been carried out for discontinuity surface  $D$  for which we distinguish "+" and "-" sides. Inclinations of axes of the extreme principal stresses  $T_1$  and  $T_3$  ( $T_1 > T_3$ , tension is positive) from both sides are different but should be potentially compatible which means existence of stress magnitudes ensuring equilibrium. To check the compatibility of axes at any point, they should be orthogonally projected in a special way onto the plane tangent to the surface. This yields the so-called shear sectors  $S^+$  and  $S^-$  containing all the possible shear stress direction vectors  $\mathbf{p}^+$  and  $\mathbf{p}^-$ ,  $|\mathbf{p}^+| = |\mathbf{p}^-| = 1$ , at a local point. The  $T_1$  and  $T_3$  axes from both sides are potentially compatible if and only if the intersection of  $S^-$  and  $S^+$  is nonempty. Indeed, if the intersection is empty the necessary condition of equilibrium,  $\mathbf{p}^- = \mathbf{p}^+$ , cannot be fulfilled. Let now the  $S^-$  sector with the central angle  $\varphi^-$  be given at any point on the surface  $D$ . Then possible orientations of the  $T_1$  and  $T_3$  axes at this point from the "+" side can be presented by the stress orientation sphere which is divided into 3 pairs of areas: tension, compression and compression-tension. Tension areas can not contain poles of the  $T_3$  axes whereas poles of the  $T_1$  axes are prohibited for the compression areas. The areas of compression-tension, which are spherical digons with angle  $\varphi^-$ , may contain the  $T_1$  or  $T_3$  pole but not both poles simultaneously. In some particular cases, when the vector  $\mathbf{p}^-$  has a unique direction ( $\varphi^- = 0$ ), the compression-tension digons disappear and the stress orientation sphere degenerates into a "beach ball" diagram which is characteristic for the earthquake focal mechanism solutions. If the potentially compatible pairs of the  $T_1$  and  $T_3$  axes are given, then the jumps of the stress values across the surface  $D$  can be easily calculated.

The results obtained have a wide range of applications. For example, no trihedron of the principal stress axes can be associated with a slip along already existing fault. In fact, 6 values are to be determined. These values define orientation of the potentially compatible stress axes on both sides of the fault. Existing restrictions are too weak to ensure that these values can be determined locally.