



## Convective instability of sludge storage under evaporation and solar radiation

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The sludge storages are an important part of production cycle at salt manufacturing, water supply, etc. A quality of water in the storage depends on mixing of pure water and settled sediment. One of the leading factors is thermal convection. There are two main mechanisms of the layer instability exist. First, it is instability of water due to evaporation from the free surface [1]. It cools the water from upside, increases the particles concentration and leads to the instability in the near-surface layer. Second, the sediment absorbs a solar radiation and heats the liquid from below making it unstable in the near-bottom area.

We assume the initial state is the mechanical equilibrium. The water and sediment particles are motionless, the sediment forms a uniform sludge layer of thickness  $z_0$ , there are no evaporation and heating by solar energy, and the temperature has a linear profile is determined by fixed upper and bottom temperatures of the layer. Taking into account the evaporation and solar radiation absorption, we obtain a non-stationary solution for the temperature using Fourier series method. The local temperature gradients increases rapidly with time, and local Rayleigh number can be estimated by thermal conduction length  $L_t$ :

$$Ra_{loc}(z, t) = \frac{g\beta(\partial T(z, t)/\partial z)L_t^4}{\nu\chi}, \quad L_t \sim \sqrt{\chi t}, \quad (1)$$

where  $g$  is gravity acceleration,  $\beta$ ,  $\nu$  and  $\chi$  are thermal volume expansion coefficient, kinematic viscosity and thermal conductivity of the liquid, respectively.  $Ra_{loc}^*$  reaches the critical value at finite time  $t^*$  and water motion begins.

The maximal power of solar radiation in visible band equals  $230 \text{ Wt/m}^2$  at the latitude of "Uralkali" salt manufacturer (Berezniki, Perm Region, Russian Federation). We neglect IR and UV radiation because of its huge absorption by water [2]. The evaporation speed is found using results for shallow water reservoir [3] and meteorological data for Berezniki [4]. We get the  $t^* \sim 6 \cdot 10^2 \text{ s}$  (10 min) for the layer of 1 m depth and  $t^* \sim 2 \cdot 10^3 \text{ s}$  (40 min) for the layer of 10 m depth.

Dynamic of the system is studied by the Galerkin–Kantorovich method. Using the follow basis along  $z$ -axis:

$$w_n = \cos q_n z - \cot q_n \sinh q_n z - \cosh q_n z + \coth q_n \sinh q_n z, \quad \tan q_n = \tanh q_n, \quad (2)$$

$$t_n = \sin p_n z, \quad p_n = \frac{\pi}{2}(2n - 1), \quad n = 1, 2, 3 \dots, \quad (3)$$

we introduce an infinite family of low-mode approximations of the full model. We found the parameter deviations from initial state grow rapidly with  $Ra > 0$  and oscillate with  $Ra < 0$  at the lowest order. Here,  $Ra$  is defined by temperature difference between upper and bottom sides of the layer under pure evaporation. The lowest order model does not describe the system in full, because the unstable areas are localized within layer.

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