



Space-Time Modelling of Groundwater Level Using Spartan Covariance Function

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Geostatistical models often need to handle variables that change in space and in time, such as the groundwater level of aquifers. A major advantage of space-time observations is that a higher number of data supports parameter estimation and prediction. In a statistical context, space-time data can be considered as realizations of random fields that are spatially extended and evolve in time. The combination of spatial and temporal measurements in sparsely monitored watersheds can provide very useful information by incorporating spatiotemporal correlations. Spatiotemporal interpolation is usually performed by applying the standard Kriging algorithms extended in a space-time framework. Spatiotemporal covariance functions for groundwater level modelling, however, have not been widely developed.

We present a new non-separable theoretical spatiotemporal variogram function which is based on the Spartan covariance family and evaluate its performance in spatiotemporal Kriging (STRK) interpolation. The original spatial expression (Hristopulos and Elogne 2007) that has been successfully used for the spatial interpolation of groundwater level (Varouchakis and Hristopulos 2013) is modified by defining the following space-time normalized distance $h = \sqrt{h_r^2 + \alpha h_\tau^2}$, $h_r = \frac{r}{\xi_r}$, $h_\tau = \frac{\tau}{\xi_\tau}$; where r is the spatial lag vector, τ the temporal lag vector, ξ_r is the correlation length in position space (r) and ξ_τ in time (τ), h the normalized space-time lag vector, $h = |h|$ is its Euclidean norm of the normalized space-time lag and α the coefficient that determines the relative weight of the time lag.

The space-time experimental semivariogram is determined from the biannual (wet and dry period) time series of groundwater level residuals (obtained from the original series after trend removal) between the years 1981 and 2003 at ten sampling stations located in the Mires hydrological basin in the island of Crete (Greece). After the hydrological year 2002-2003 there is a significant groundwater level increase during the wet period of 2003-2004 and a considerable drop during the dry period of 2005-2006. Both periods are associated with significant annual changes in the precipitation compared to the basin average, i.e., a 40% increase and 65% decrease, respectively.

We use STRK to “predict” the groundwater level for the two selected hydrological periods (wet period of 2003-2004 and dry period of 2005-2006) at each sampling station. The predictions are validated using the respective measured values. The novel Spartan spatiotemporal covariance function gives a mean absolute relative prediction error of 12%. This is 45% lower than the respective value obtained with the commonly used product-sum covariance function, and 31% lower than the respective value obtained with a non-separable function based on the diffusion equation (Kolovos et al. 2010). The advantage of the Spartan space-time covariance model is confirmed with statistical measures such as the root mean square standardized error (RMSSE), the modified coefficient of model efficiency, E' (Legates and McCabe, 1999) and the modified Index of Agreement, IoA' (Janssen and Heuberger, 1995).

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