



## **Multilevel Monte Carlo for Two Phase Flow and Transport in a Subsurface Reservoir with Random Permeability**

Florian Müller, Patrick Jenny, and Meyer Daniel

Institute of Fluid Dynamics, ETH Zürich, Zürich, Switzerland

To a large extent, the flow and transport behaviour within a subsurface reservoir is governed by its permeability. Typically, permeability measurements of a subsurface reservoir are affordable at few spatial locations only. Due to this lack of information, permeability fields are preferably described by stochastic models rather than deterministically. A stochastic method is needed to assess the transition of the input uncertainty in permeability through the system of partial differential equations describing flow and transport to the output quantity of interest.

Monte Carlo (MC) is an established method for quantifying uncertainty arising in subsurface flow and transport problems. Although robust and easy to implement, MC suffers from slow statistical convergence. To reduce the computational cost of MC, the multilevel Monte Carlo (MLMC) method was introduced. Instead of sampling a random output quantity of interest on the finest affordable grid as in case of MC, MLMC operates on a hierarchy of grids. If parts of the sampling process are successfully delegated to coarser grids where sampling is inexpensive, MLMC can dramatically outperform MC. MLMC has proven to accelerate MC for several applications including integration problems, stochastic ordinary differential equations in finance as well as stochastic elliptic and hyperbolic partial differential equations.

In this study, MLMC is combined with a reservoir simulator to assess uncertain two phase (water/oil) flow and transport within a random permeability field. The performance of MLMC is compared to MC for a two-dimensional reservoir with a multi-point Gaussian logarithmic permeability field. It is found that MLMC yields significant speed-ups with respect to MC while providing results of essentially equal accuracy. This finding holds true not only for one specific Gaussian logarithmic permeability model but for a range of correlation lengths and variances.