



Covariance Models for Hydrological Applications

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This methodological contribution aims to present some new covariance models with applications in the stochastic analysis of hydrological processes. More specifically, we present explicit expressions for radially symmetric, non-differentiable, Spartan covariance functions in one, two, and three dimensions.

The Spartan covariance parameters include a characteristic length, an amplitude coefficient, and a rigidity coefficient which determines the shape of the covariance function. Different expressions are obtained depending on the value of the rigidity coefficient and the dimensionality. If the value of the rigidity coefficient is much larger than one, the Spartan covariance function exhibits multiscaling. Spartan covariance models are more flexible than the classical geostatistical models (e.g., spherical, exponential). Their non-differentiability makes them suitable for modelling the properties of geological media.

We also present a family of radially symmetric, infinitely differentiable Bessel-Lommel covariance functions which are valid in any dimension. These models involve combinations of Bessel and Lommel functions. They provide a generalization of the J-Bessel covariance function, and they can be used to model smooth processes with an oscillatory decay of correlations.

We discuss the dependence of the integral range of the Spartan and Bessel-Lommel covariance functions on the parameters. We point out that the dependence is not uniquely specified by the characteristic length, unlike the classical geostatistical models.

Finally, we define and discuss the use of the generalized spectrum for characterizing different correlation length scales; the spectrum is defined in terms of an exponent α . We show that the spectrum values obtained for exponent values less than one can be used to discriminate between mean-square continuous but non-differentiable random fields.

References

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