



A stochastic perturbation theory for non-autonomous systems

Woosok Moon (1) and John Wettlaufer (2,3)

(1) University of Cambridge, DAMTP, Cambridge, United Kingdom (wm275@cam.ac.uk), (2) Mathematical Institute, University of Oxford, Oxford OX2 6GG, UK, (3) Yale University, New Haven, CT, 06520-8109, USA

We develop a perturbation theory for a class of first order nonlinear non-autonomous stochastic ordinary differential equations that arise in climate physics. The perturbative procedure produces moments in terms of integral delay equations, whose order by order decay is characterized in a Floquet-like sense. Both additive and multiplicative sources of noise are discussed and the question of how the nature of the noise influences the results is addressed theoretically and numerically. By invoking the Martingale property, we rationalize the transformation of the underlying Stratonovich form of the model to an Ito form, independent of whether the noise is additive or multiplicative. The generality of the analysis is demonstrated by developing it both for a Brownian particle moving in a periodically forced quartic potential, which acts as a simple model of stochastic resonance, as well as for our more complex climate physics model. The validity of the approach is shown by comparison with numerical solutions. The particular climate dynamics problem upon which we focus involves a low-order model for the evolution of Arctic sea ice under the influence of increasing greenhouse gas forcing ΔF_0 . The deterministic model, developed by Eisenman and Wettlaufer EW09 exhibits several transitions as ΔF_0 increases and the stochastic analysis is used to understand the manner in which noise influences these transitions and the stability of the system.

Eisenman, I., and J. S. Wettlaufer, "Nonlinear threshold behavior during the loss of Arctic sea ice," *Proc. Natl. Acad. Sci. USA*, 106, 28–32, 2009.