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Fast algorithm for calculation of the moving tsunami wave height

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One of the most urgent problems of mathematical tsunami modeling is estimation of a tsunami wave height while a wave approaches to the coastal zone. There are two methods for solving this problem, namely, Airy-Green formula in one-dimensional case

$$S(x) = S(0)\sqrt[4]{H(0)/H(x)},$$

and numerical solution of an initial-boundary value problem for linear shallow water equations

$$\begin{cases} \eta_{tt} = \operatorname{div}(gH(x,y)\operatorname{grad}\eta), & (x,y,t)\in\Omega_T := \Omega\times(0,T);\\ \eta\big|_{t=0} = q(x,y), \quad \eta_t\big|_{t=0} = 0, & (x,y)\in\Omega := (0,L_x)\times(0,L_y);\\ \eta\big|_{\partial\Omega_T} = 0. \end{cases} \tag{1}$$

Here $\eta(x,y,t)$ is the free water surface vertical displacement, $H(x,y)$ is the depth at point (x,y), $q(x,y)$ is the initial amplitude of a tsunami wave, $S(x)$ is a moving tsunami wave height at point x. The main difficulty problem of tsunami modeling is a very big size of the computational domain Ω_T. The calculation of the function $\eta(x,y,t)$ of three variables in Ω_T requires large computing resources. We construct a new algorithm to solve numerically the problem of determining the moving tsunami wave height which is based on kinematic-type approach and analytical representation of fundamental solution (2).

The wave is supposed to be generated by the seismic fault of the bottom $\eta(x,y,0) = g(y)\cdot\theta(x)$, where $\theta(x)$ is a Heaviside theta-function. Let $\tau(x,y)$ be a solution of the eikonal equation

$$\tau_x^2 + \tau_y^2 = \frac{1}{gH(x,y)},$$

satisfying initial conditions $\tau(0,y) = 0$ and $\tau_x(0,y) = (gH(0,y))^{-1/2}$.

Introducing new variables and new functions:

$$z = \tau(x,y), \quad u(z,y,t) = \eta_t(x,y,t),\ b(z,y) = \sqrt{gH(x,y)}.$$

We obtain an initial-boundary value problem in new variables from (1)

$$\begin{cases} u_{tt} = u_{zz} + b^2u_{yy} + 2b^2\tau_y u_{zy} + \left(b^2(\tau_{xx}+\tau_{yy}) + 2\frac{b_z}{b} + 2bb_y\tau_y\right)u_z + \\ \qquad +2b(b_z\tau_y + b_y)u_y, \qquad z,y>0, t>0,\\ u\big|_{t<0} = 0, \quad u_z\big|_{z=0} = -g(y)(b^{-2}(0,y) - \tau_y^2(0,y))^{-1/2}\delta(t), \quad y>0, t>0. \end{cases}$$

Then after some mathematical transformation we get the structure of the function $u(x,y,t)$ in the form

$$u(z,y,t) = S(z,y)\cdot\theta(t-z) + \tilde{u}(z,y,t). \tag{2}$$

Here $\tilde{u}(z,y,t)$ is a smooth function, $S(z,y)$ is the solution of the problem:

$$\begin{cases} S_z + b^2\tau_y S_y + \left(\frac{1}{2}b^2(\tau_{xx}+\tau_{yy}) + \frac{b_z}{b} + bb_y\tau_y\right)S = 0, & z,y>0,\\ S(0,y) = \frac{g(y)}{2}\left(b^{-2}(0,y) - \tau_y^2(0,y)\right)^{-1/2}, & y>0. \end{cases} \tag{3}$$

Note that the problem (3) is two-dimensional which allows one to reduce the number of operations in 1.5 times. The algorithm makes it possible to calculate the moving tsunami wave height $S(z,y)$ coming to a given point (z_0,y_0) as well as the arrival time.

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