



An Idealized Model of Plant and Soil Dynamics

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Following wildfire events the landscape commonly becomes denuded of vegetation cover, resulting in systems prone to soil loss and degradation. In this context soil dynamics are an intricate process balanced between pedogenesis, which is a relatively slow process and erosion which depends on many inert (e.g. soil texture, slope, precipitation and wind) and biological factors such as vegetation properties, grazing intensity, and human disturbance.

We develop a simple homogenous, spatially implicit, theoretical model of the global dynamics of the interactions between vegetation and soil using a system of two nonlinear differential equations describing this interdependence, assuming a double feedback between them - plants control erosion and soil availability facilitates plants growth:

$$\frac{dV}{dt} = rV \left(K - \frac{K}{1 + aS} - V \right) \quad (1)$$

$$\frac{dS}{dt} = \sigma - \varepsilon S e^{-cV} \quad (2)$$

where V and S represent vegetation cover and soil availability, respectively. Vegetation growth is similar to the classical logistic model with a growth rate of $r(\text{yr}^{-1})$, however, the "carrying capacity" (K) is dependent on soil availability (a^1 is the amount of soil where V is reduced by half). Soil influxes at a constant rate $\sigma(\text{mm}\cdot\text{yr}^{-1})$ and is eroded at a constant rate $\varepsilon(\text{yr}^{-1})$, while vegetation abates this process modeled as a decreasing exponent as the effectiveness of vegetation in reducing soil erosion (c). Parameter values were chosen from a variable range found in the literature: $r=0.01 \text{ yr}^{-1}$, $K=75\%$, $a^1=1$, $\sigma=1 \text{ mm}\cdot\text{yr}^{-1}$, $\varepsilon=0.1 \text{ yr}^{-1}$, $c=0.08$.

Complex properties emerge from this model. At certain parameter values ($cK \leq 4$) the model predicts one of two steady states – full recovery of vegetation cover or a degraded barren system. However, at certain boundary conditions ($cK > 4$ and $\lambda_1 \leq \sigma/\varepsilon \leq \lambda_2$, see Article for terms of λ_1 and λ_2) bistability may be observed. We also show that erosion seems to be the determining factor in this system, and we identify the threshold values from which beyond the systems become unstable.

The model predicts that certain ecosystems will be highly stable in one of two states, while others might be bistable transitioning between these two states through perturbations. This is an indicator of hysteresis, possibly indicating the ability of the system to shift leading to sudden and dramatic changes; formalizing the conceptual model shown by Davenport et al. (1998) and others. Following the establishment of these interrelationships, the role of repeated disturbances, such as wildfires, was assessed with numerical analysis in determining the long term dynamics of coupled soil-vegetation systems.