



To $b = 1$ or not to $b = 1$. Numerical, conceptual, hydraulic and geometric explanations for observed streamflow recession behaviour — a case of being right for which reason?

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Recession discharge from hillslopes and catchments is commonly summarized by the top-down Brutsaert and Nieber (1977) analysis in which a power law of the form $-dQ/dt = aQ^b$ is fitted through recession data. In many cases exponent b is found to be within the range 1 to 2. A key question in hillslope and catchment hydrology is how this range can be explained from underlying bottom-up physical theory and system properties.

A common approach in hillslope hydrology is to apply the Boussinesq equation, either in its original nonlinear form, or a linearized simplification, in concert with assumptions like thin soils of uniform hydraulic conductivity.

We found that the nonlinear Boussinesq equation in this setting leads to $b = 0$, and thus is inconsistent with observations. Careless interpretation of the recession response from a Boussinesq model could lead to an erroneous conclusion of $b = 1$. We demonstrate how this artifactual model response arises from the internal numerics of spatially distributed PDE models that hinder complete drying out.

We demonstrate how this trait — models that can't dry out — by necessity lead to $b \geq 1$ behaviour. Some commonly used model approaches share this trait:

- As described above, numerical implementations of the nonlinear Boussinesq equation retain the last bits of water, and therefore suggest $b = 1$ (which is shown to be an artifact)
- Both analytical and numerical solutions to the linearized Boussinesq equation are unable to move the drainage front downhill (as explained earlier by Stagnitti et al. (2004)), which causes retainment of water, leading to $b = 1$ at all times.
- Vertically decreasing hydraulic conductivity, e.g. a power-law or exponential profile, leads to $b = 1$ to 2.

Based on the reasoning that the linearized Boussinesq equation (as a meta-model) is only valid if it adequately mimics the essential dynamics of the nonlinear Boussinesq equation (as a reference model) we conclude that explanations of observed $b = 1$ based on the linearized Boussinesq equation are less likely to be valid.

This further suggests that observed $b = 1$ to 2 from sloping aquifers is more likely to be due to system properties, most probably vertically decreasing conductivity, planform divergence and profile convexity, all of which have a positive effect on b .