Hydroelastic analysis of ice shelves under long wave excitation

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1. Introduction and Research Motivation

Ice sheets act as climate controllers, regulating temperature and ocean current circulation, with profound effects on global weather patterns and climatic change. The stability of ice shelves extending into the ocean is greatly influenced by ocean wave forcing.

The acknowledgement of the adverse effects of climate change has motivated scientific research in the field of ocean wave-ice interaction, as wave trains are bound to become rougher while elevated temperatures weaken ice formations.

The calving of the Sulzberger Ice Shelf, Antartica, in 11 March 2011, induced by the Honsu earthquake and subsequent Tsunami, showed evidence of the detrimental impact of tsunami excitation on a previously stable ice formation [1].

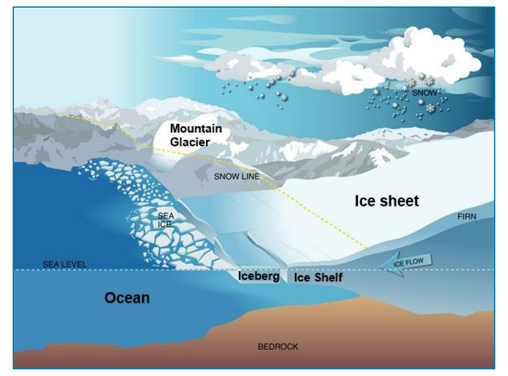


Figure 1.1 : Ice shelf schematic (Source :http://earthobservatory.nasa.gov/NaturalHazards)

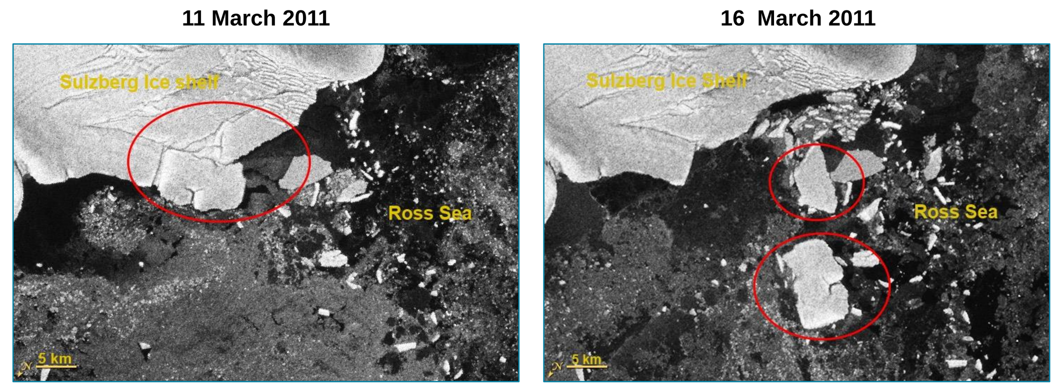


Figure 1.2 : ASAR images of the Sulzberger Ice shelf, before and after the 2011 tsunami (Source : http://earthobservatory.nasa.gov/NaturalHazards)

- ❑ Our research focuses on the transient response of ice shelves under long wave excitation. Notably, similar analysis applies to the study of Very Large Floating Structures [2].
- ❑ Our aim is to employ the finite element method in order to develop a computational tool for the determination of stress fields, induced by long waves, in ice shelves.

2. Governing Equations

❖ Consider an **elastic, heterogeneous, thin plate of length *L* fixed on one end**, and resting on an inviscid, incompressible fluid layer over impermeable bottom. The plate represents the ice shelf extending into the ocean (*see fig. 1.1*), subjected to water wave induced forcing.

❖ Initial – Boundary Value Hydroelastic problem

$\nabla^2\varphi = 0$ in Ω

$\nabla\varphi\cdot\mathbf{n} = 0$ on Γ_b

$\varphi_{tt} + g\varphi_z = 0$ on Γ_f

$m\eta_{tt} + (D\eta_{xx})_{xx} = q + p$ on Γ_p

$\varphi_x = 0\ (x\to\infty)$ on Γ_i

$H(x)$: Bathymetry function
$d(x)$: Thickness function
$\eta(x,t)$: Upper surface elevation/plate deflection
$\varphi(x,z,t)$: Velocity potential
$m(x)$: Plate mass per unit length
$D(x)$: Flexural rigidity of the plate
$q(x,t)$: Plate load
g : Acceleration of gravity

Initial excitation: $\eta(x,0)=\eta_0(x)$

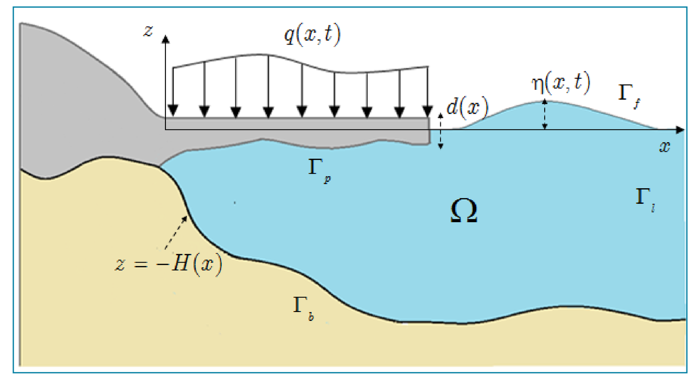


Figure 2.1 : Schematic diagram of the general 2-D initial-boundary value problem (extends infinitely in y direction)

Long Wave approximation $\quad \dfrac{\text{wavelength}}{\text{water depth}} \gg 1$

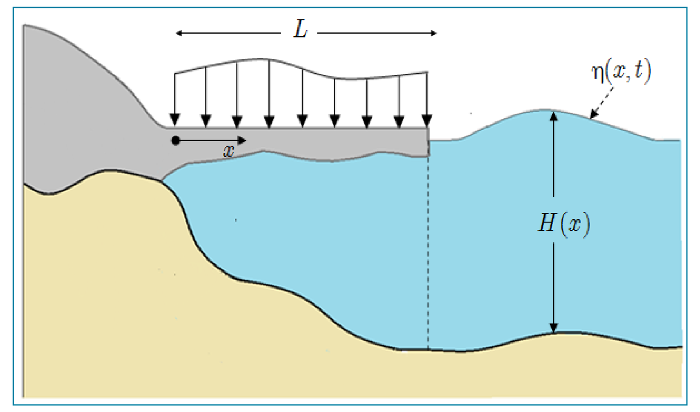


Figure 2.2 : Schematic diagram of the reduced shallow water problem

❖ Introducing the nondimensional quantities

$$\tilde{x}=x/L,\ \tilde{\eta}=\eta/L,\ \tilde{t}=t\sqrt{gL^{-1}},\ \tilde{\varphi}_0=g^{-1/2}L^{-3/2}\varphi_0,\ \tilde{\varphi}_1=g^{-1/2}L^{-3/2}\varphi_1$$

and dropping tildes:

$$M\eta_{tt}+(K\eta_{xx})_{xx}+\eta+\varphi_{0t}=Q\ ,\ x\in S_0=(0,1) \qquad (2.1)$$

$$\eta_t+(B\varphi_{0x})_x=0\ ,\ x\in S_0=(0,1) \qquad (2.2)$$

$$\varphi_{1tt}-(B\varphi_{1x})_x=0,\ x\in S_1=(1,+\infty) \qquad (2.3)$$

$M(x)=\dfrac{m(x)}{\rho_w L}$, $K(x)=\dfrac{D(x)}{\rho_w g L^4}$, $B(x)=\dfrac{H(x)}{L}$, $Q(x,t)=\dfrac{q(x,t)}{\rho_w g L}$, $S(x)=\dfrac{\eta_0(x)}{L}$

φ_0 : Velocity Potential in S_0
φ_1 : Velocity Potential in S_1
ρ_w : Water density

Boundary conditions:	Interface conditions:	Initial conditions:
$\eta(0,t)=\eta_x(0,t)=0$	$B(1^-)\varphi_{0x}(1^-,t)$	$\eta(x,0)=\eta_t(x,0)=\varphi_{0t}(x,0)=0$, in S_0
$\eta_{xx}(1,t)=\eta_{xxx}(1,t)=0$	$=B(1^+)\varphi_{1x}(1^+,t)$	$\varphi_{1x}(x,0)=0,\ \varphi_{1t}(x,0)=-S(x)$, in S_1
$\varphi_{0x}(0,t)=0$	$\varphi_{0t}(1^-,t)=\varphi_{1t}(1^+,t)$	

Bending Moment: $M_b=K\eta_{xx}$

Shear Force: $V=(K\eta_{xx})_x$

3. Finite Elements

❖ In order to derive the variational formulation, equations (2.1), (2.2), (2.3) are multiplied by the weight functions , $v\in H^2(S_0): v(0)=v_x(0)=0$, $w_0\in H^1(S_0)\ w_1\in H^1(S_1)$ respectively and integrated over their domain.

$$\int_0^1 Mv\eta_{tt}dx+\int_0^1 Kv_{xx}\eta_{xx}dx+\int_0^1 v\eta dx+\int_0^1 v\varphi_{0t}dx=\int_0^1 vQ(x,t)dx \qquad (3.1)$$

$$\int_0^1 w_0\eta_t dx+\left[Bw_0\varphi_{0x}\right]_0^1-\int_0^1 Bw_{0x}\varphi_{0x}dx=0 \qquad (3.2)$$

$$\int_1^\infty w_1\varphi_{1tt}dx-\left[Bw_1\varphi_{1x}\right]_1^\infty+\int_1^\infty Bw_{1x}\varphi_{1x}dx=0 \qquad (3.3)$$

❖ A special 3-node hydroelastic element (HELFEM) is developed and employed for domain S_0 , whereas quadratic Lagrange elements are used in S_1

- ✓ C^1 Hermite cubic shape functions are used as basis for the interpolation of the plate displacement.
- ✓ C^0 Lagrange quadratic shape functions are used for the interpolation of the fluid velocity potential function.

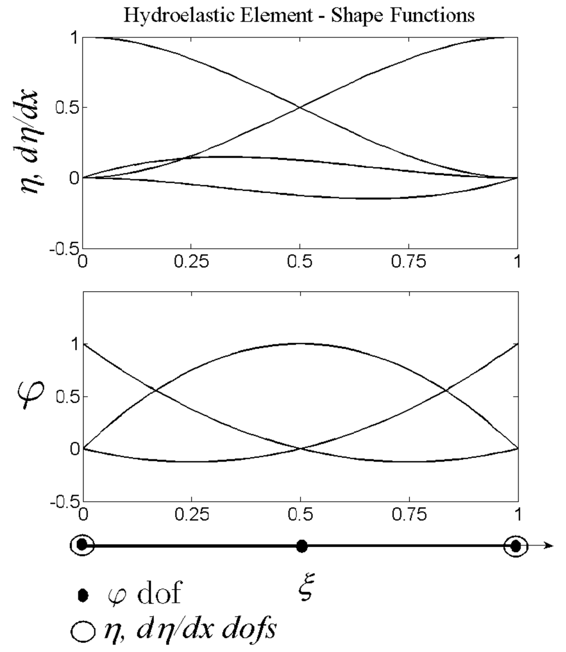


Figure 3.1: Hydroelastic Element-HELFEM

❖The modal expansion solution for the transient hydroelastic response of a heterogeneous, freely floating plate, over shallow water of variable depth, given in Sturova (2009) [3], was compared with the finite element solution.

✓ Time Integration of the resulting ODE system $\mathbf{M}u_{tt}+\mathbf{C}u_t+\mathbf{K}u=F$ (where u is the vector of nodal unknowns) is performed with the Newmark algorithm [4]

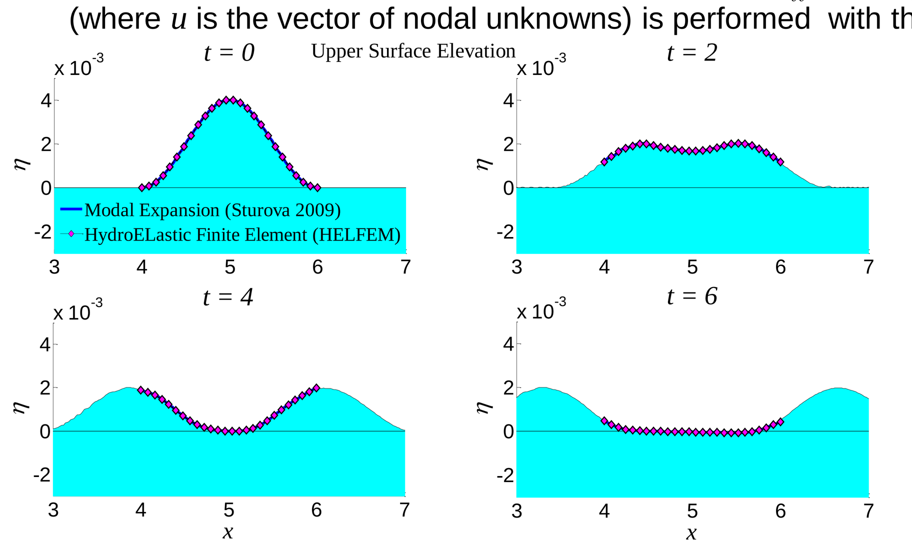


Figure 3.2: A comparison between the method presented in [2] and the finite element solution, in a benchmark example (Sturova (2009)) is shown.
An initial plate deformation/ upper surface elevation of the form:
$\eta_0=0.5+0.5\cos(x-5)$, $4\le x\le 6$
Is prescribed.
Excellent agreement was found between the two methods, thus establishing the convergence of the finite element solution.

4. Results

❖The transient response of the fixed plate, subject to an incident long wave is shown. Two configurations are chosen.

✓Configuration A features a plate of constant thickness.

✓Configuration B features a plate of variable thickness.

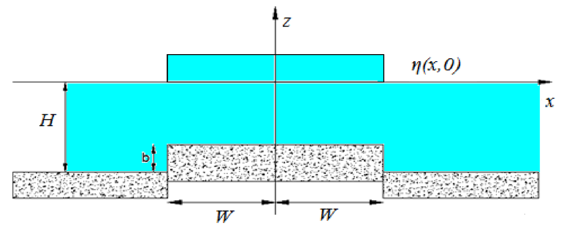


Figure 4.2: Initial water elevation due to seabed dislocation (Hammack(1973) [5], Okal and Synolakis (2003) [6]).

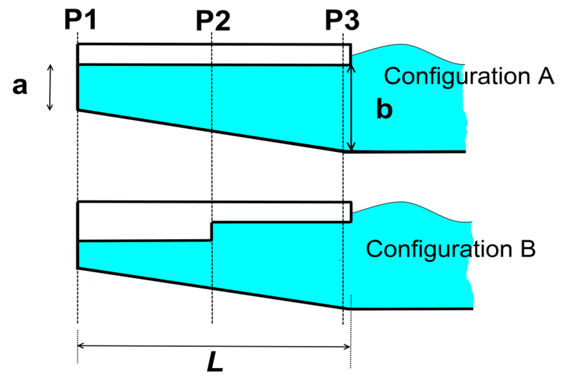


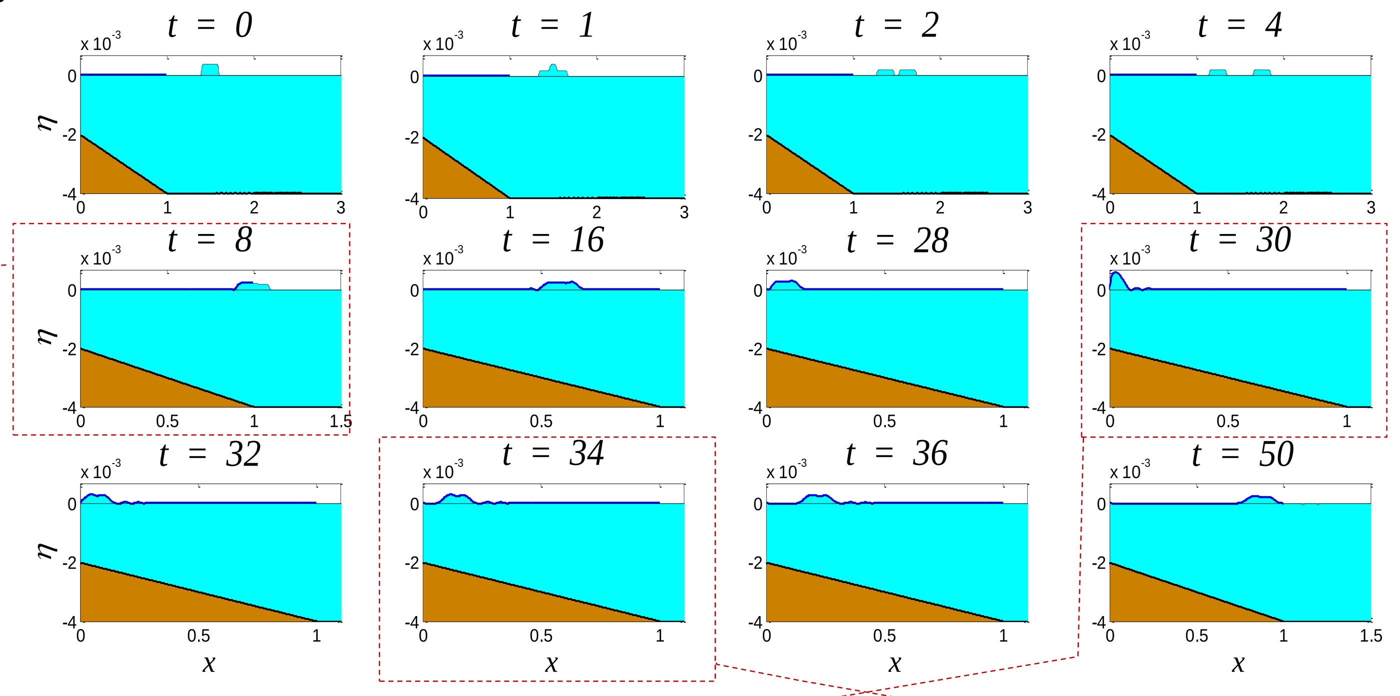
Figure 4.1: Schematic representation of configurations A and B.

❖ For the present analysis, seabed dislocation is considered to be the mechanism of Tsunami generation.

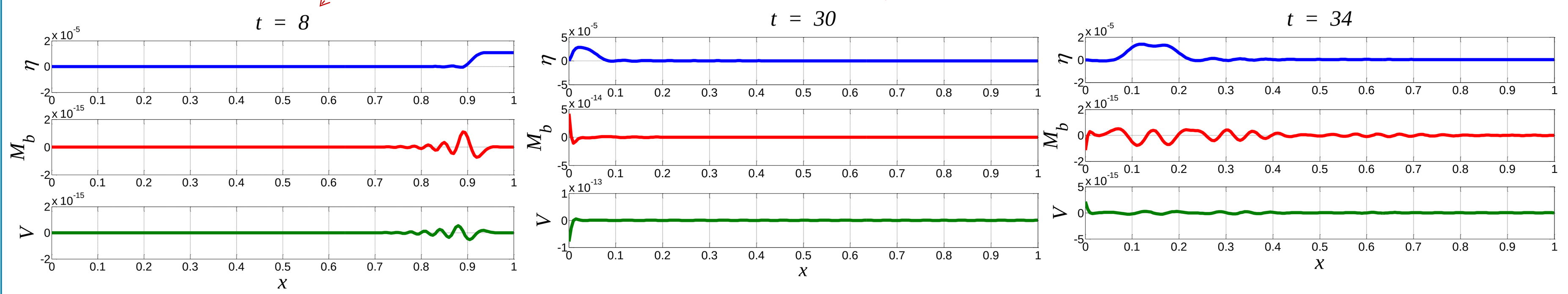
Configuration A

❖ Approximating the Sulzberger ice shelf front [1] the length of the plate is set to 100 km. The constant plate thickness is assumed to be 100 m, while the maximum depth is b=400 m. Initial excitation was provided in the form of a seabed dislocation induced tsunami. Wavelength of the initial pulse is 20 km, while amplitude is 2 m. Seabed topography is defined by a ratio a/b=0.5

Figure 4.3: FEM solution for the upper surface elevation-displacement. Three snapshots are accompanied by the corresponding bending moment and shear force distributions.
(Upper Surface elevation is exaggerated)



The initial upper surface displacement forms two identical travelling waves, propagating at opposite directions, with constant speed and featuring zero dispersion. The wave that impacts the free end of the plate is partially reflected while it continues to propagate, passing under the plate and providing the initial flexural excitation. The hydroelastic wave appears to be dispersive as expected. When reaching the fixed edge (x=0), the wave is fully reflected and subsequently propagates away from the plate origin. The theoretical maximum absolute bending moment and shear force are observed at the vicinity of the fixed edge.



❖ A parametric study of the effects of bottom topography on the extreme values of plate deflection, bending moment and shear force was carried out. Extreme values increase as water depth under the plate decreases.

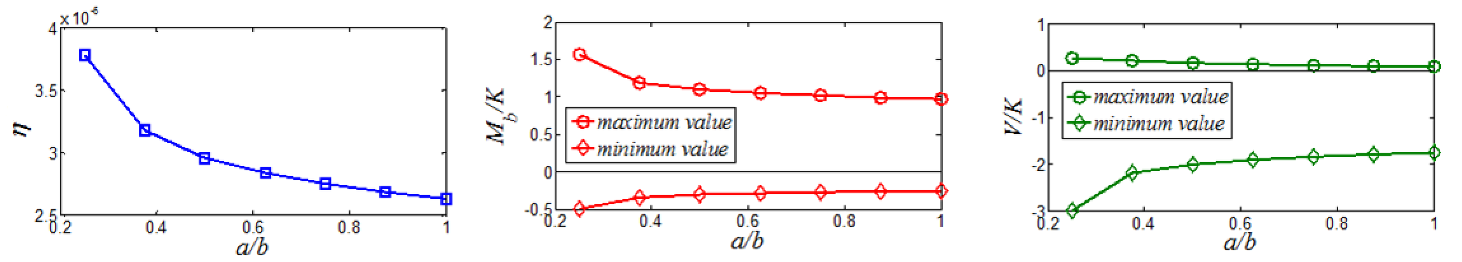
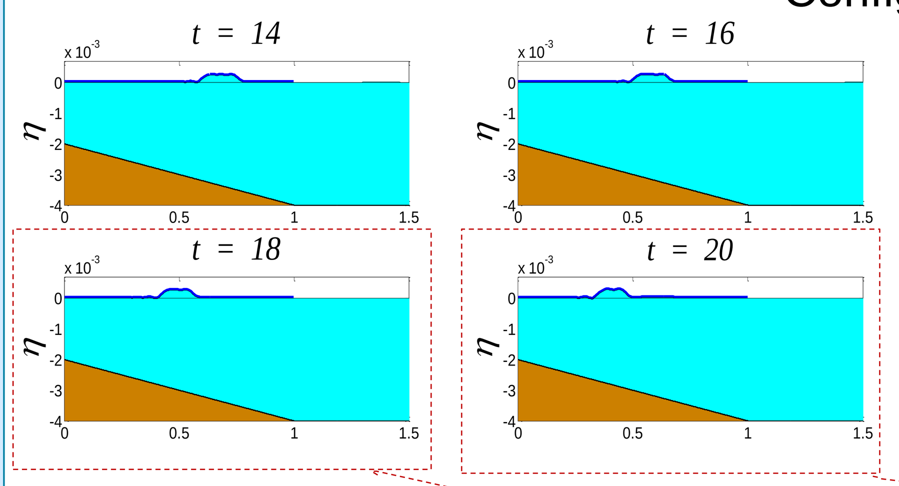


Figure 4.4 : Parametric study of the bottom topography (slope steepness) variation. Extreme plate deflection, bending moment and shear force values increase as the ratio ***a/b*** decreases.

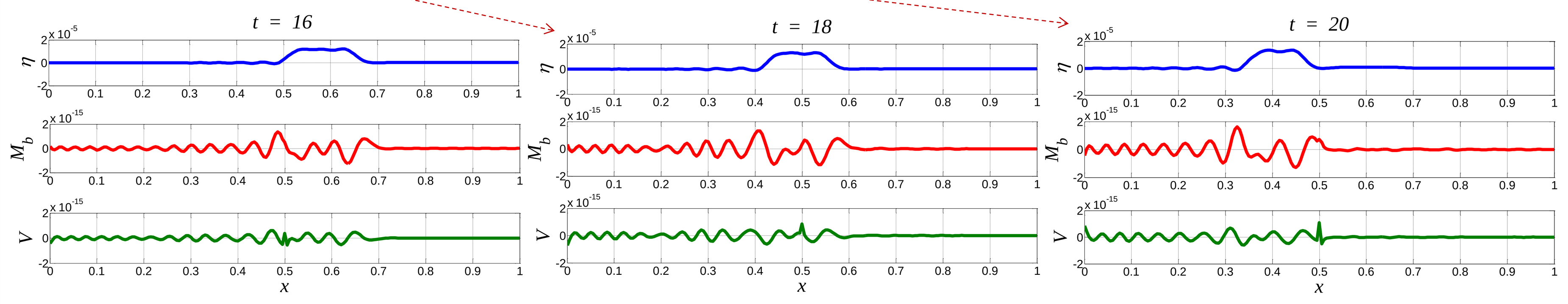
Configuration B



❖ In order to examine the effects of the ice shelf cross section variation, a plate featuring a stepped thickness (150 m thickness from its fixed edge up to its midpoint and 100 m from the midpoint to the free end) is studied.

Figure 4.5: FEM solution for the upper surface elevation-displacement. Three snapshots are accompanied by the corresponding bending moment and shear force horizontal distributions.

Due to the thickness variation, the propagating pulse exhibits kinks at point P2 ,that are particularly evident in the case of shear force distributions



❖ The extrema of bending moment and shear force spatial distribution for configurations A and B as functions of time are compared.

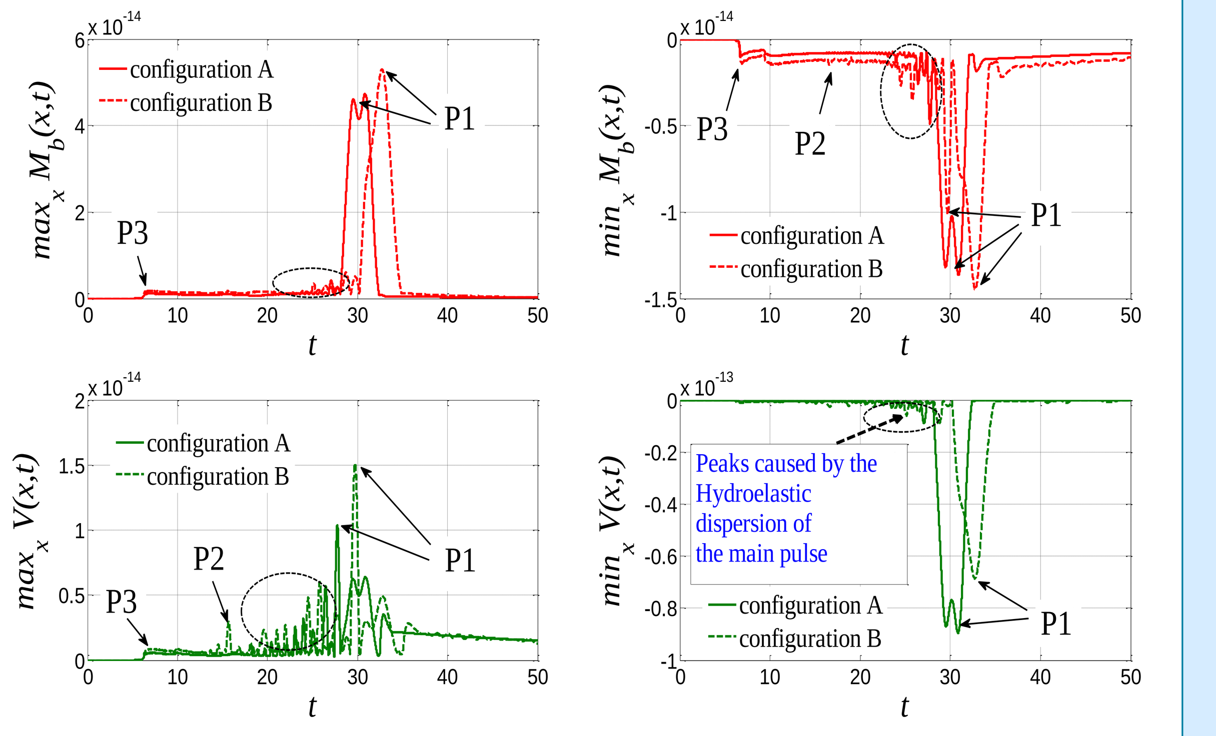


Figure 4.6 : Comparison of bending moment and shear force maximum/minimum values against time, for configurations A and B.

At point P1 (fixed edge), the positive bending moment is greater for configuration B. In all cases, it is evident that the hydroelastic dispersion of the main pulse is intensified for configuration B, after point P2. The dispersion of the hydroelastic wave manifest itself (see ***figure 4.6)*** through the formation of escalating peaks, regarding the bending moment and shear force values at P1, preceding the arrival of the main pulse at the same point. Due to the cross-sectional variation, configuration B features a sudden increase of maximum shear force at point P2 for t□15.5.

5. Conclusions and Further research

The hydroelastic response of an ice shelf under long wave excitation is analysed by means of the Finite Element method. Deflection values, bending moment and shear force distributions are calculated for two ice shelf profiles and different bathymetry functions. Bending moment and shear force extrema were observed at the fixed edge of the plate. Additionally, a parametric study verified a direct correlation between decreasing depths and increasing extrema, as the ice self continental end is approached.

Future research directions include:

- ❑ Study of defect/crack propagation due to hydroelastic forcing. Determination of the stress intensity Factor using simple beam models [7] .
- ❑ Extension of the methodology with application to short waves [8].

References

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