Travel time inversion by Bayesian Inferential Statistics

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We are presenting a fully Bayesian approach in inferential statistics determining the posterior probability distribution for non-directly observable quantities (e.g. Earth model parameters). In contrast to deterministic methods established in Geophysics we are following a probabilistic approach focusing on exploring variabilities and uncertainties of model parameters. The aim of that work will be to quantify how well a parameter is determined by a priori knowledge (e.g. existing earth models, rock properties) completed by data obtained from multiple sources (e.g. seismograms, well logs, outcrops).

Therefore we are considering the system in question as a Gaussian random process. As a consequence, the randomness in that system is completely determined by its mean and covariance function. Our prior knowledge of that conceptional random process is represented by its a priori mean. The associated covariance function – which quantifies the statistical correlation at two different points – forms the basis of rules for interpolating values at points for which there are no observations.

Measurements are assumed to be linear (or linearized) observational functionals of the system. The desired quantities we want to predict are thought to be linear functionals, too. Due to linearity, observables and predictions are Gaussian distributed as well depending on the prior mean and covariance function. The presented approach calculates the prediction’s conditional probability distribution posterior to a set of measurements. That conditional probability combines observations and prior knowledge yielding the posterior distribution, an updated distribution for the predictions.

As proof of concept, the estimation of propagation velocities in a simple travel-time model is presented. The two observational functionals considered are travel-times and point measures of the velocity model. The system in question is the underlying velocity model itself, assumed as Gaussian random process. Therefore, a rough guess of the model serves as prior mean and the long-tailed stationary Cauchy kernel is used as co-variance function. Obviously, predictions ought to be the underlying velocity model at arbitrary points (e.g. a regularly spaced grid). Subsequently, we are calculate the velocity model’s conditional probability distribution posterior to the set of observations. Although the predicted mean value is the most probable velocity model, its variability which is determined by the posterior variance is much more expressive. This gives us the opportunity to quantify how well the inverted velocity model is determined.

A consecutive extension of the given example is to adopt our approach to 3d seismic wave propagation. Seismological records together with well logs will form the data basis. Prior knowledge are existing Earth models completed by guessing uncertainties. As result the algorithm will yield the model’s PDE covering the uncertainties in elastic parameters.