



Critical Concavity of a Drainage Basin for Steady-State

Jongmin Byun and Kyungrock Paik

School of Civil, Environmental, and Architectural Engineering, Korea University

Longitudinal profiles of natural streams are known to show concave forms. Saying A as drainage area, channel gradient S can be expressed as the power-law, $S \propto A^{-\theta}$ (Flint, 1974), which is one of the scale-invariant features of drainage basin. According to literature, θ of most natural streams falls into a narrow range ($0.4 < \theta < 0.7$) (Tucker and Whipple, 2002). It leads to fundamental questions: “Why does θ falls into such narrow range?” and “How is this related with other power-law scaling relationships reported in natural drainage basins?”

To answer above questions, we analytically derive θ for a steady-state drainage basin following Lane’s equilibrium (Lane, 1955) throughout the corridor and named this specific case as the ‘critical concavity’. In the derivation, sediment transport capacity is estimated by unit stream power model (Yang, 1976), yielding a power function of upstream area. Stability of channel at a local point occurs when incoming flux equals outgoing flux at the point. Therefore, given the drainage at steady-state where all channel beds are stable, the exponent of the power function should be zero. From this, we can determine the critical concavity. Considering ranges of variables associated in this derivation, critical concavity cannot be resolved as a single definite value, rather a range of critical concavity is suggested. This range well agrees with the widely reported range of θ ($0.4 < \theta < 0.7$) in natural streams.

In this theoretical study, inter-relationships between power-laws such as hydraulic geometry (Leopold and Maddock, 1953), dominant discharge-drainage area (Knighton *et al.*, 1999), and concavity, are coupled into the power-law framework of stream power sediment transport model. This allows us to explore close relationships between their power-law exponents: their relative roles and sensitivity. Detailed analysis and implications will be presented.

References

- Flint, J. J., 1974, Stream gradient as a function of order, magnitude, and discharge, *Water Resources Research*, 10, 969-973.
- Knighton, A. D., 1999, Downstream variation in stream power, *Geomorphology*, 29, 293-306.
- Lane, E. W., 1955, The importance of fluvial morphology in hydraulic engineering, *American Society of Civil Engineers, Proceedings*, 81, 1-17
- Leopold, L. B., Maddock, T., 1953, The hydraulic geometry of stream channels and some physiographic implications, United States Government Printing Office, 1953.
- Tucker, G. E., Whipple, K. X., 2002, Topographic outcomes predicted by stream erosion models: Sensitivity analysis and intermodel comparison, *Journal of Geophysical Research*, 107(B9), 2179, doi:10.1029/2001JB000162, 2002.
- Yang, C. T., 1976, Minimum unit stream power and fluvial hydraulics, *Journal of Hydraulics Division, ASCE* 102, 919-934.