



Filtered density functions for solutes transported in heterogeneous aquifers

Lennart Schüler (1,2), Nicolae Suciu (3,4), Sabine Attinger (1,2), and Peter Knabner (3)

(1) Friedrich Schiller University Jena, Institute for Geoscience, Germany (lennart.schueler@ufz.de, sabine.atinger@ufz.de),
(2) UFZ-Helmholtz Centre for Environmental Research, Department of Computational Environmental Systems, Germany, (3)
Friedrich-Alexander University of Erlangen-Nuremberg, Mathematics Department, Germany (suciu@math.fau.de,
knabner@math.fau.de), (4) Romanian Academy, Tiberiu Popoviciu Institute of Numerical Analysis

Geological formations are heterogeneous. Their properties are usually not measurable everywhere but only at some locations. This lack of knowledge implies uncertainty in aquifer parameters like hydraulic conductivity. As a consequence, transport of solutes through these formations is also uncertain and has to be described in a probabilistic sense. Mean and variance estimates of solute concentrations give some information on the probability distribution of concentrations.

We present analytical results for the concentration variance in lowest order perturbation theory. Going beyond mean and variance estimates, we are able to state the transport equation for the whole probability density distribution of the concentration by adopting an approach first introduced in turbulence theory by Colucci et al. (1998), Phys. Fluids. However, we propose an alternative closure strategy and compare resulting effects on the probability density distribution.

One prominent result is the temporal behavior of the solute concentration variance and its increase with distance from the injection point or with travel time. The generation of variance is balanced in the long time limit by a term that eliminates concentration variance. This balance indicates that the solute concentration is well described by its mean in the ergodic limit. In other words, only in the very long time limit the mean concentration is a good descriptor for the concentration. Hence, it is not sufficient to only use the mean to accurately describe the transport of a solute for earlier times.

We propose to make use of spatial filtering and filtered density functions (FDF) to avoid these errors for finite times. The FDF evolution equations are similar in form with those of probability density functions, but their solutions are still random and depend on the width of the spatial filter. The filter is adjusted in order to minimize the produced concentration variance and thus the error introduced by averaging is minimized too.