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A discontinuous Galerkin method for studying elasticity and variable viscosity Stokes problems

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Traditionally in the geodynamics community, staggered grid finite difference schemes and mixed Finite Elements (FE) have been utilised to discretise the variable viscosity (VV) Stokes problem. These methods have been demonstrated to be sufficiently robust and accurate for a wide range of variable viscosity problems involving both smooth viscosity structures possessing large spatial variations, and for discontinuous viscosity structures. One caveat of the aforementioned discretisations is that they tend to have inf-sup constants which are highly dependent on the cell aspect ratio. Whilst high order mixed FE approaches utilising spaces defined via $Q_k - Q_{k-2}$, $k \geq 3$, alleviate this shortcoming, such elements are seldomly used as they are computationally expensive, the definition of multi-level preconditioners is complex, and spectral accuracy is often not obtained.

Discontinuous Galerkin (DG) methods offer the advantage that spaces can be constructed which have both low order in velocity and pressure and inf-sup constants which are not sensitive to the element aspect ratio. To date, DG discretisations have not been extensively used within geodynamic applications associated with VV Stokes formulations.

Here we rigorously evaluate the applicability of two Interior Penalty Discontinuous Galerkin methods, namely the Nonsymmetric and Symmetric Interior Penalty Galerkin methods (NIPG and SIPG) for compressible elasticity and incompressible, variable viscosity Stokes problems. Through a suite of numerical experiments, our evaluation considers the stability, order of accuracy and robustness of the NIPG and SIPG discretisations for cases with both smooth and discontinuous coefficients.

Using manufactured solutions, we confirm that both DG formulations are stable and result in convergent solutions for displacement based elasticity formulations, even in the limit of Poisson ratio approaching 0.5. With regards to incompressible flow simulations, using the analytic solution SolCx, we (i) show that the SIPG method yields optimal order of accuracy for Stokes and linear elasticity problems with both smooth and discontinuous coefficients with both Q_k and P_k spaces and (ii) confirm that the NIPG method produces optimal order of accuracy (w.r.t velocity errors) when the polynomial degree of the velocity space is even, and suboptimal accuracy (one order lower) when the polynomial degree is odd. We also have established that the well known "water-glass" test is satisfied and that buoyancy driven flows (such as Rayleigh-Taylor instabilities) do not result in any spurious features in either the pressure or velocity field. Lastly, we will discuss appropriate preconditioners for the Stokes DG discretisation and provide some preliminary results from time-dependent, multi-layer viscous buckling simulations.